A UNIFIED APPROACH TO PATIENT CLASSIFICATION AND NURSE STAFFING FOR LONG-TERM CARE FACILITIES

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A Unified Approach to Patient Classification and Nurse Staffing for Long-Term Care Facilities

by

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ABSTRACT

In order to assist administrators of long-term facilities in responding to the changing environment of and demand for institutional care of the elderly and chronically infirm, a unified approach to patient classification and nurse staffing has been developed. The approach is essentially structure oriented and is directed toward the presentation of alternative nurse staffing strategies; its basis is the quantification of patient nursing needs through intermediate steps of assessment and classification. The classification system, in turn, has been related to the demand for nursing services, as categorized by care area, for each of three homogeneous patient groupings. Effective allocation and assignment of patient nursing care activities have been modeled by use of techniques of mathematical programming.

The ability to predict, with reasonable confidence, the level of care to which a patient would most appropriately be assigned based on an assessment of various functioning status items, psycho-social status indicators, and medically defined conditions was considered initially. Application of an existing nonlinear multiple regression technique to patient assessment data drawn from a comprehensive assessment

instrument yielded a means by which patients could be classified according to 37 health status indicators. The regression method was chosen for its specific applicability to problems of fitting a polychotomous, ordered response, analogous to the level of care. In order to achieve a classification system of reasonable size for use in typical long-term care facilities, various subsets of the original 37 variables were tested for their prediction and recognition powers; a subset of 12 variables was shown to possess adequate capabilities in this regard. This subset was synthesized into an implementable patient classification system procedure.

The determination of an optimal staff mix, allocation of nursing time, and assignment of nurses to patient care demands was captured in the Basic Staffing Model (BSM), a mixed-integer linear program. The BSM maximizes an objective function that combines the concepts of the appropriate assignment of nursing personnel to specific patient care activities and the satisfaction of high priority demands at greater than minimum levels, if possible. Model constraints include the availability of nursing resources, budgetary limitations, legal staffing requirements, and adherence to bounded representations of patient nursing care activities. Two extensions to the BSM, Model I and

Model II, were developed to facilitate the exploration of alternative staffing policies. Model I allows for the parametric representation of changes in the total patient-centered services provided and personnel budget, while Model II enables parametric alterations in upper and lower bounds on staff and patient demands to be examined.

Two algorithms were derived for the efficient solution of both BSM extensions. Recognizing the forms of the models, branch and bound methods incorporating post-optimality analysis in specialized linear programming routines were developed. These methods were based on the Upper Bounded Dual Simplex algorithm of Wagner; their computational characteristics for both general parametric mixed-integer linear programs and specific examples of applications to nurse staffing problems were investigated.

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To my family

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Chapter I. Introduction: The Need for Research

I.1 General Discussion

As the twenty-first century approaches, one of the most important components of our multi-billion dollar a year national health system, long-term care, seems certain to become even more vital. With age being the primary, although not exclusive, determinant of entry into the long-term care system, demographic projections indicate that potential demands will be high. Brody [12] notes that while at the turn of the century some three million persons (4% of the U.S. population) were age 65 or over, we now have approximately 20 million persons (10%) in this category. Figures cited by Flagle [24] show a 55% increase by 1990 in persons 65 or over, based on 1970 census data for the state of Maryland. The United Nations estimates that for the period 1970-1980 the world's aged population will grow by 38% while the total population increases by 28%.

The response of Federal and state governments to this trend has for the most part been in the form of specific provisions in the current Medicare/Medicaid legislation.* These programs provide support or

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* Titles XVIII and XIX of the Social Security Act.

supplementary assistance to those persons in need of long-term care and who meet certain eligibility criteria. Still being debated is proposed legislation establishing a National Health Insurance (NHI) system which presumably will contain payment structures for some forms of care of the chronically ill. Flagle [24] notes that the five-year period following the installation of Medicare saw a 68% increase in nursing home patients. If this is any indication of the underlying elasticity of demand for long-term care service, the potential impact of NHI could be highly significant, especially if payments are weighted towards institutional care rather than possible alternative care programs. The relative magnitude of this impact can be estimated by noting figures prepared by the Department of Health, Education, and Welfare (HEW) [75] showing that there are currently some 16,000 long-term care facilities in the U.S. caring for about 1.1 million persons, representing approximately 5.2% of the population over 65.

Because of the large amount of federal funds being devoted to health care in general, and with the probable expenditure of much larger sums in the future, the need for accountability and cost-benefit analysis has become evident. The legislative reaction has been the creation of Professional Standards Review Organizations

(PSRO) on a state-wide basis to set standards and criteria for the various component specialties of the health delivery system, and to monitor compliance with such guidelines. The enabling legislation* specifically calls for such standards review organizations in the long-term care area, although to date little progress has been made towards complying with this mandate. It is certain, however, that the decisions ultimately made will bear heavily on both the forms of long-term care delivery and the quality of care provided.

The potential impact of population increases, current and proposed publicly-supported health insurance systems, and PSRO taken both individually and in concert have given rise to problems that must be dealt with by national and regional policy-makers. It is incumbent upon them to predict the resource requirements generated by these factors in a number of alternative care settings and to choose the mix of services which will most adequately serve the client population. Assessment of the social and technological requirements for prolonging patient independence in home care settings must be carried out and weighed against the alternative of institutionalization. Secondary effects such as those to be felt

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* 1972 Amendments to the Social Security Act.

in the labor and education markets for health professionals must be recognized. Where increased utilization of institutions is projected, plans must be made to increase existing capacity either by direct government involvement or through incentives to the private sector. In short, there is much work to be done on the "macro" planning level before the predictions become reality.

I.2 The Need for Research at the Facility Level.

It should be obvious at this point that regardless of the increasing emphasis on alternative settings for long-term care, the future portends greater requirements for quantity and quality of care to be provided by facilities. Being licensees of their respective state governments, with additional federal certification required in order to care for Medicare patients, individual facilities must be capable of reacting in a positive way to the requirements set forth by regulatory agencies. In doing so, they will in a sense be translating to a "micro" level those broad objectives mentioned earlier. This dissertation represents an effort towards assisting the individual facility in this task.

As an historical note the reliance upon an institutional setting for care of the aged and chronically infirm is a relatively recent phenomenon. Brody [12]

identifies six distinct periods in the history of long-term care in the U.S. The colonial period and the period of the 19th and early 20th centuries to 1920 were characterized by care provided in the home, with relatively little dependence on locally-financed public almshouses. Between 1920 and 1935 the rise of the eleomosynary institution and the creation of publicly-financed care systems took place, after which the years 1935-1945 constituted a time of reaction to such developments. The postwar years to 1965 saw the increased entry of privately-owned proprietary nursing homes into the LTC system, while concurrently, the major mechanisms for public and private support were being molded. The advent of the current period is marked by the enactment of Titles XVIII (Medicare) and XIX (Medicaid) of the Social Security Act of 1965, from which the concepts of a stratified LTC system responsive to patient needs are derived.

Since we shall later have cause to quantify the relationships between patient health status and the levels of care of the stratified system, we pause to clarify some definitions. Because most of this study was conducted in the State of Maryland, definitions of LTC facilities are provided below as taken from that State's Report of the Governor's Commission on Nursing Homes [56, p. 117]; roughly analogous interpretations

are given in other sources (cf. Coggeshall [16]). Note that to a large extent, the levels of care correspond to the type of facility involved, and we shall therefore use the terminology interchangeably.*

<u>Chronic Disease Hospital</u>: A facility that provides medical care to persons incapacitated by illness, with the intent and, indeed, mandate to rehabilitate and return them to the community.

<u>Skilled Nursing Home</u>: A facility that maintains facilities and staff necessary to render skilled nursing care (which means by an RN or by an LPN under supervision).

<u>Intermediate A Facility</u>: A facility that provides long-term care to residents whose illness is not acute and whose proper care requires no more than 8 hours per day under supervision of an RN or LPN.

<u>Intermediate B Facility</u>: A facility that provides services which a person normally provides himself, but for which he is now dependent upon others because of advanced age, physical, or mental limitations.

Current practice calls for the physical placement of a

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* More detailed discussion of specific regulations pertaining to levels of care will be provided later.

long-term care patient into a facility certified to provide that level of care which is deemed appropriate for his or her needs. A change in patient requirements thus necessitates movement from one facility to another, although recent trends indicate an increase in facilities with multi-level certifications, thereby eliminating this requirement.

With this background information in mind, we must necessarily consider the task of the facility administrator, upon whom the responsibility for compliance with regulatory agencies rests. In broad terms, the administrator must be concerned with the adequate and timely provision of the three components of institutional life as identified by Brody [12]:

1. Basic maintenance services (shelter, food, sanitation);
2. Medical and paramedical services to foster maximum physical health and functioning capacities;
3. Psychosocial components.

The second and third of these components are highly labor-intensive, and their provision accounts for the major portion of LTC facility operating costs. Estimates provided by the Governor's Commission [56], Schwartz [63], and HEW [75] put total labor costs at about

55%-60% of total operating budgets. The above studies indicate that payroll and fringe benefit costs attributable to nurse staffing make up anywhere from 35% to 50% of operating costs. In short, the administrator is confronted with a budgeting problem which is certain to involve relatively large sums to be spent on nursing staff.

Cost, however, is neither the only nor the most important consideration in developing a nurse staffing program. Aydelotte [5] points out five additional factors which must be weighed and acted upon in order that an acceptable program may be obtained:

1. Quality of patient care to be delivered and its measurement;
2. Characteristics of the patients and their care requirements;
3. Prediction of the supply of nursing manpower required for (1) and (2);
4. Logistics of the staffing program and its control;
5. Evaluation of the quality of care desired, thereby measuring the success of the staffing itself.

Item (2) is perhaps the key factor among all five, since unless a reasonably accurate appraisal of the care needs of facility residents can be made, the staffing mix and

staffing patterns developed in concert with budget considerations will be at least partially ineffective, with a consequent degradation in quality of care.

What means, then, are available to administrators and their directors of nursing for relating patient health status and the implied care needs to a nurse staffing system? Relying upon the placement status of the patient as an assessment tool allows the use of minimum staffing levels as specified by state regulations, for example, as a guide. The number of hours of bedside care per patient of each level, staff-to-patient ratios, and supervisory personnel required could be determined for the patient mix that exists, and the minimum or near minimum resources maintained. If further funds are available, some choice could be made as to additional personnel required. Patient placement status, however, may not necessarily be an accurate reflection of demand for nursing services, since many other intervening factors are involved in the placement decision. Information generated in the course of utilization review might give a more accurate picture of patient care needs, but relating such information to nursing requirements and hence to budgeting is likely to be difficult and to occur in an untimely fashion.

The need for maximization of care quality for a

given budgetary level, and the relationship between patient health status and nurse staffing are thus central issues confronting the administrator. The evaluation of quality of care suggested in item (5) above is a most difficult task. What can be done at present is to either assume quality is acceptable with the staffing pattern employed or to rely on some appropriate measure of quality surrogates. In the former case, a staffing pattern which contributes to job satisfaction by assigning nurses to tasks commensurate with their training can lead to better staff morale. Revans [57,58] has demonstrated that staff morale is a good indicator of the quality of care provided. In connection with the use of quality of care surrogates, Burroughs [14] showed that a number of easily measured indicators (e.g., number of requests for pain medication, number of incontinent patients, length of stay, etc.) could be quantitatively related to subjective estimates of quality of care by use of discriminant analysis. These studies were conducted in acute care facilities, however, and additional analysis is needed in the LTC area.

I.3 The Proposed Research

In order to provide administrators and nursing directors, in their roles as decision makers, with useful

tools for assessing the relative merit of alternative courses of action, a unified approach is proposed. This methodology is based on a multi-disciplinary operations research point of view and will address the following four issues:

1. Budgetary restrictions;
2. Assessment of patient care needs;
3. Nurse staff allocation; and
4. Quality of care.

It will be shown that a key factor in such an approach is relating patient health status, through classification, with nursing care requirements.

After an initial review of the existing literature pertaining to the general problems of patient classification and nurse staffing, a classification system for long-term care patients into levels of care will be developed. The system will initially take as its input information available from a comprehensive patient assessment instrument. It will be shown that a reasonably accurate level of care classification can be derived from components of the assessment instrument by application of a regression-based statistical technique. A proposed classification instrument designed for easy use by facility staff will be demonstrated.

We shall then forge the link between patient

assessment and classification, and demand for nursing services, using information thereby obtained as inputs into a mathematical programming model designed to synthesize the four elements mentioned above. Certain theoretical properties of the model form will be advanced to better allow for examination of alternatives. Such advances are made in the area of parametric mixed-integer linear programming.

The proposed staffing model takes as its objective to be maximized the allocation of nursing care activities to the most appropriate nursing skill level available and a priority measure on care area/classification combinations. By invoking such a discipline, good quality care should be achievable. In order to assure that the model is computationally tractable, a grouping of the nursing tasks into various "care areas" will be utilized. Attention is given to the varying needs of patients for the tasks of each care area by basing the model on a subdivision of the care areas by the levels of care indicated through patient classification. Constraints on personnel budget and availability, legal staffing minimums, and unit aggregate demands by care area and patient classification are also included. In addition, the latter of these three constraint sets includes a priority related system for meeting demands.

Through the use of mathematical programming methods, staffing levels and skill level mix as well as allocation of direct care demands by skill level will be examined under a variety of circumstances. An example of how such information may be generated and presented will be provided. Finally, we will conclude by noting difficulties encountered in the course of the research, conclusions reached, and recommendations for further study.

Chapter II. A Review of Existing Methodologies for Patient Classification and Nursing Resource Allocation

II.1 Introduction

In order to place the current study into proper perspective, we briefly review some of the major studies most closely related to our twin objectives of patient classification and nursing resource allocation. The summaries presented are in no way meant to be an exhaustive survey of preceding work; rather, they represent an effort to capture the essence of the extensive body of significant literature in the field.

Studies involving classification, and the concomitant nurse staffing problems of allocation and assignment in an acute care setting will be discussed first, since historically they provide the conceptual basis for subsequent work in long-term care. An examination of work specifically devoted to this latter area will follow. Further references and amplification of earlier research may be found in the surveys conducted by Aydelotte [5], Stimson and Stimson [71], and Shuman and Wolfe [67].

II.2 Patient Classification in an Acute Care Setting

Connor [18]

It can be argued that most patient classification procedures in use today are based on the fundamental concepts proposed by Connor in his pioneering studies of nursing unit staffing requirements. Essentially, the aim of his study was to determine the underlying functional relationship between patient care needs and the response to these needs in terms of nursing manpower. If patients can be assessed and their care needs anticipated, a more flexible staffing pattern is possible than the commonly used ratios based primarily on census counts. As an initial step the amount of direct care provided to a sample patient population was measured. It was found that the time devoted to direct care for individual patients by the nursing staff varied with the degree of self-sufficiency of the patient, i.e., the more self-sufficient the patient, the less direct care he received. Moreover, it was found that the patient population could be classified into essentially three groups: 1) a self-care group at one extreme consisting of patients who were able to take care of most of their needs and made little or no demand on nursing staff, 2) a total-care group at the other extreme consisting of patients who were often seriously ill or bedfast and made large demands on nursing

staff for such care activities as feeding, bathing, etc., and 3) an intermediate care group who required only partial assistance from the nursing staff. The three groupings were verified by the emergence of a tri-modal distribution of measured direct care times, as shown in Figure II.1

A classification scheme was developed, using a Boolean logic, that classified patients into self, partial, and total care categories on the basis of those factors that reflected their degree of self-sufficiency. Indeed, it was these factors that correlated with the large demands made on nursing staff in the provision of direct care time. The basic classification scheme, and the factors involved, is shown in Figure II.2.

Connor then determined the average number of hours of direct care devoted to patients in each of the three patient categories by noting the group averages for the three populations as shown in Figure II.1. After determining the average number of hours devoted to indirect care by additional work sampling on typical medical-surgical units, Connor proposed that the total nursing hours for a given patient mix on an acute care unit could be predicated by first computing a Direct Care Index

$$I = .5N_1 + 1.0N_2 + 2.5N_3 \qquad \text{(II.1)}$$

Figure II.1

Distribution of Minutes of Direct Patient Care Rendered as Found by Connor

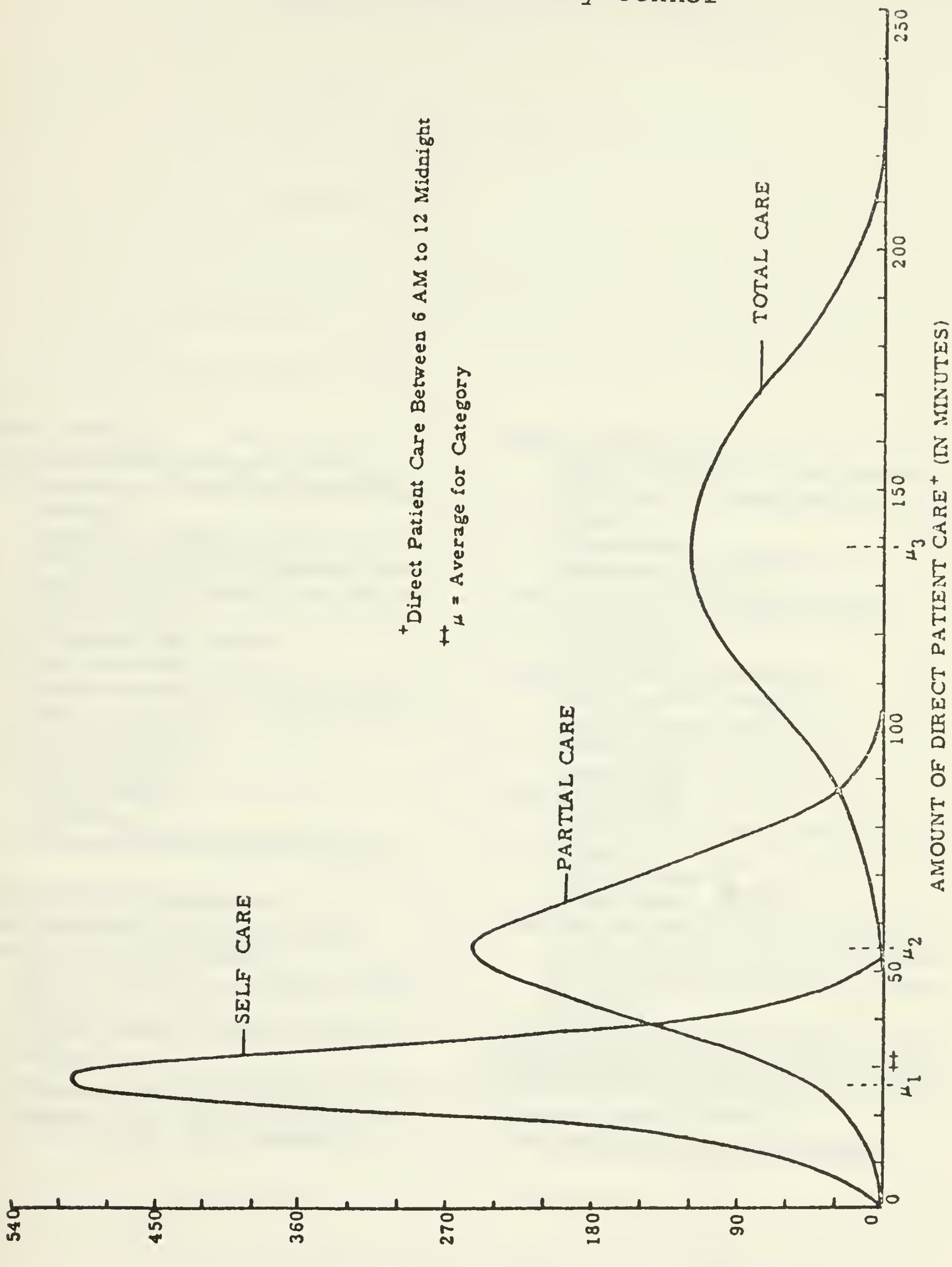

Figure II.2

Connor's Combinations of Factors for Categorization of Patients

Category I. Self-care

Any of the following combinations checked

(a) Ambulatory, or up in chair—Self (without assistance)
  Feeding self, or requires food cut
  Bathing in bathroom, or at bedside—Partial Self (can bathe self except for back and perhaps extremities)

(b) Ambulatory—with assistance
  Up in chair—Self
  Bathing in bathroom, or at bedside—Partial Self

(c) As in (a and b) with
  Vision inadequate
  Oxygen therapy
  I.V. feeding
  but no two of these factors simultaneously

Category II. Partial or Intermediate care

Any of the following combinations checked

(a) Ambulatory—with assistance
  Bathing in bathroom, or at bedside—Partial Self
  Feeding—Complete Assistance (except I.V. feeding)
  Vision inadequate, Oxygen therapy } optional (does not affect classification under these conditions)

(b) Up in chair—Self
  Bathing at bedside—Complete Assistance
  Feeding Self, or requires food cut or I.V. feeding
  Oxygen therapy, Vision inadequate } optional

(c) As in (b) with the following changes
  Up in chair—With Assistance
  Bath at bedside

(d) Up in chair—With Assistance
  Bath at bedside—Partial Self
  Feeding—Complete Assistance
  Vision inadequate, Oxygen therapy } optional

(e) Bath at bedside
  Feeding—Self, or requires food cut, or I.V. feeding
  Vision inadequate, Oxygen therapy } optional

(f) Being Specialed of Necessity (patient has continuous nursing assistance to the extent that meal relief must be provided for special duty nurse)

NOTE: Any patient who otherwise falls into Categories I or II, but who is under suction therapy or is in isolation, incontinent (including wound drainage necessitating change of bed linens), or markedly emotionally disturbed (needs almost constant observation, in single room, creates disturbance) will be dropped to the next category.

Category III. Intensive, or "total", care

All combinations not previously mentioned.

where

N_1 = number of patients classified as Category I.

N_2 = number of patients classified as Category II,

N_3 = number of patients classified as Category III,

and the coefficients of the equation represent the care hours required for each category. The total work load can then be found from

$$WL = I + 20 \text{ hours} \qquad \text{(II.2)}$$

where the additional factor of 20 hours represents the average time required for all other indirect care activities. In a subsequent extension, a method for "controlled variable staffing" based on predictions from the above indexes was demonstrated by noting that the daily averages and shortfalls in nursing time demanded, relative to the unit's basic staff, could be controlled through use of additional float personnel. Young [83] and Wolfe and Young [81] provide further insights on the practical aspects of the Connor staffing approach in their discussion of day-to-day implementation procedures. Although essentially sound and more effective than traditional techniques used for staffing, the method has drawbacks in that it neither indicates the nursing skill levels required nor suggests how available time might be allocated

between patients of various categories.

Young [84]

It might be anticipated that a patient classification system, such as that proposed by Connor, might not be universally applicable and should therefore be applied with some caution. Young, in a study of the direct care provided to pediatric patients found that, although pediatric patients can be classified into age groupings, no classification as to degree of self-sufficiency within such groupings was possible. Indeed, as opposed to adult patients, the more self-sufficient a pediatric patient became, the greater the demand on nursing time. Examination of data on direct care nursing time revealed a continuous distribution of times with a mean of about 90 minutes; i.e., the frequency distribution of direct care times was unimodal indicating no distinct subpopulations. In order to determine total minutes of direct care required it was felt that a multiple linear regression model was most appropriate, using as the regressors the 0-1 variables corresponding to the subcategorization of 12 factors determined by professional nurses. These are shown in Table II.1.

The numbers in the right-hand column are not to be interpreted as time required for the factors indicated; they are, in fact, regression coefficients. However, for

Table II.1

Young's Classification of Pediatric Patients

FACTOR	DEGREE	Regression Coefficient for Determining Direct Care Times
MOBILITY:	Complete Self	1
	With Assistance	10
	Bedfast	25
ACTIVITY:	Restrained	20
	Hyperactive	10
	Emotionally Disturbed	10
	Normal Activity	10
	Lethargic	1
	Unconscious	5
FEEDING:	Bottles x 6	5
	x 8	10
	x 12	15
	Feed Complete Assistance	25
	Feed Partial Assistance	5
	Feed Gavage	5
	Self	1
I.V.:	Normal (Simple)	1
	Special (Complicated)	2
MED. FREQ.:	Average	1
	Frequently	20
TREATMENT:	Oxygen	20
	Dressing-Compresses	50
	Suction	20
	Tracheotomy	20
	Drainage	15
	Croupette	15
	Pediatric Procedures	20
INCONTINENT:	Diapers Frequently (10,20,30 per day)	1
	Bed Change	10
TPR:	Normal Frequency	1
	Special	10
ISOLATION:	Protective	3
	Preventive	3
FAMILY	Assist	-10
MEMBERS:	Non-Assist	1
	Special Attention Given	2
SPECIAL OF	Special Nurse	2
NEC:	Relief Only	1
BATHING:	Partial Assistance	10
	Complete Assistance	15

Add constant of 40 to total

those factors checked for a particular patient, the sum of the numbers, when added to a constant of 40 does provide the total direct care time, in minutes, required for that patient. Young found a very high correlation between these predicted times and observed times. More significantly, it was again found that patient assessment and, in this instance, "continuous" classification was much more predictive of patient care required than a census count and rule-of-thumb.

Other Studies

For the sake of completeness we briefly note some additional studies involving hospital inpatient classification. The National League of Nursing Education [48], Cloussen [15], Stanford [70], and the Hospital Association of New York State (HANYS) [30] all have proposed classification systems based on assessment of a patient's physical and emotional needs as well as the incidence and/or frequency of various medication or treatment regimens. Work by Poland, English, Thronton, and Owens [54] also involves the use of patient classification; however, since the primary thrust of the study is staffing, we defer its consideration to the next section.

II.3 Nurse Staffing in an Acute Care Setting

Beginning with the concepts proposed by Connor, the study of the nurse staffing problem has intensified with an ever-increasing degree of sophistication in the models proposed. Typically, solutions were sought for the following three aspects of staffing as they relate to demand for service:

1. Number and skill levels of nursing personnel needed;
2. The distribution of the time of available personnel among direct and indirect care activities (the "allocation" problem);
3. The delegation, by skill level, of the constituent tasks of direct and indirect care (the "assignment" problem).

Each of the methods presented below represents an attempt to answer one or more of these questions.

<u>Wolfe [80], Wolfe and Young [81,82]</u>

In order to overcome some of the shortcomings of Connor's staffing models, Wolfe proposed a methodology based on the concept of a "multiple assignment model." This approach permits the assignment of a skill level to more than one activity as opposed to the classical assignment model which imposes one-to-one assignments. Wolfe viewed all nursing activities in both the direct

and indirect care categories as falling into one of 12 mutually exclusive task complexes as shown in Table II.2. A work sampling study was undertaken to determine the nursing time required for each complex during an eight-hour day. These times were related to patient condition by regressing them on the number of patients in each of Connor's three classifications on the unit. The resulting regression coefficients, b_j, were interpreted as time required for the task complex per patient of category N_i, i=1,2, or 3; certain coefficients were found to be insignificantly different from zero and thus constant for all or a subset of the classifications.

A cost, c_{ij}, indicating both salary cost and imputed monetary disutility incurred by assigning skill level i to task complex j, was then determined. The latter cost component was obtained by application of psychometric measurement techniques to judgments solicited from both nurses and patients. Wolfe's integer programming multiple assignment model is, therefore,

$$\text{MIN} \sum_i \sum_j (c_{ij} - \alpha_i b_i) x_{ij} + \sum_i a_i \alpha_i Y_i$$

$$\begin{aligned} \text{s.t.} \sum_i x_{ij} &= 1 , && \forall j \\ a_i Y_i - \sum_j b_j x_{ij} &\geq 0 , && \forall i \qquad \text{(II.3)} \\ x_{ij} &= 0, 1 \\ Y_i &\geq 0 \text{ and integer} \end{aligned}$$

Table II.2

Task Complexes Identified by Wolfe

1. Technical Tasks I	7. Clerical Tasks I
2. Technical Tasks II	8. Clerical Tasks II
3. Evaluation of Pt. Needs and Assignment	9. Clerical Tasks III
4. Supervising and Teaching	10. Housekeeping Duties
5. Preparatory Care I	11. Escorting and Emergency Errands
6. Preparatory Care II	12. Maintenance, Checking, and Ordering of Supplies and Equipment

where

α_i is the hourly wage of skill level i,

a_i is the number of hours worked by each member of skill level i (Wolfe assumed $a_i = 8$),

x_{ij} is the assignment variable, a value of 1 indicating the assignment of skill level i to task complex j, and

Y_i is a variable indicating the number of personnel of skill level i required.

Closer examination of (II.3) reveals that the objective function calls for the minimization of an assignment cost plus a non-productivity cost, assuming the following:

1. All task complexes must be assigned;
2. Ample personnel of all skill levels are available to meet the requirement Y_i;
3. No partial shifts are allowed;
4. No splitting of task complexes between skill levels is allowed.

Probably because of these assumptions, Wolfe claims not to have touched upon the allocation and assignment problems noted earlier, but rather to have provided a means for determining the number and skill levels of nurses required on a unit. This is the primary drawback of his methodology.

Singer [68]

Recognizing the stochastic nature of requirements for nursing care as a function of a patient's condition, Singer attempted to improve the staffing aspects of Connor's work by modeling the fluctuations in the numbers of patients in each of the three classifications in a time-dependent framework. He was able to do this by using a multiple-population, immigration-death process model, and provided good predictions of Connor's direct care index over relatively short horizons. The model did not perform as well for time periods greater than about three days. Unfortunately, few indications as to how these findings may be put into practice are given.

Jelinek [32,33]

In an effort to predict the implications of any number of several well known factors on some convenient measures of nursing activity, Jelinek proposed the use of a multiple linear regression model. A major result of such a methodology is isolation of effects through controlled experimental design. The author reports an application of this technique in regressing variables indicating the number of hours per patient-day devoted to direct care, indirect care, and non-productive activities on a set of independent variables related to patient length of stay, health status (as per Connor's classification), type

of unit, unit census, and staff size and skill level mix. Among the findings reported were decreasing returns to scale for direct care activities as a function of available staff, with a corresponding increase in nonproductive activity. The findings also emphasized the poor predictive power of patient census relative to nursing activity.

Poland, English, Thornton, and Owens [54]

The authors offer a normative model for prediction of demand for direct care by means of an attractively designed system called PETO. Through work sampling, the relationships between various criterion levels of seven patient assessment factors and the number of hours of required "physical nursing care" were determined. These times were then translated into a point scale for each criterion level. Thus, a patient would be assessed in accordance with Table II.3, then the point total converted to "physical care units" (PCU) by means of Table II.4. Finally, the unit direct care work load can be found by adding all PCU's. Based on nursing care audits, the amount of required care delivered on sample units was found to be significantly increased after installation of the system. Unfortunately, however, PETO failed to indicate how staffing by skill level as well as allocation of resources may be achieved.

Table II.3

PETO Elements of Care

Element	Description	Point Value
Diet	Feeds self without supervision, or family or parent feeds patient.	1
	Feeds self with supervision of staff.	2
	Feeds self with constant presence of staff, or gastrostomy feeding every 4 hours.	4
	Total feeding by personnel or instructing the parent, or continuous I.V., or blood transfusion.	8
	Tube feedings more frequently than every 4 hours.	12
Toileting	Toilets without supervision.	1
	Toilets with supervision, or specimen collection or uses bedpan.	2
	Up to toilet with standby supervision, or output measurement every hours, or daily colostomy irrigation.	4
	Incontinent, average output.	8
	Incontinent with diarrhea, or immediate postoperative colostomy, or urethrostomy or drainage with frequent dressing change.	12
Vital signs and measurements	Routine--daily temperature, pulse, and respiration.	1
	Vital signs every 4 hours, or night check every 4 hours.	2
	Vital signs monitored, or hypothermia, or vital signs every 2 hours.	4
	Vital signs and observation every hour, or vital signs monitored, plus	

Table II.3, cont'd.

Element	Description	Point Value
	hypothermia and neurological evaluation.	8
	Blood pressure, pulse, respiration, and neurological evaluation every 30 minutes.	12
Respiratory aids	Bedside humidifier, or blow bottle.	1
	Mist or humidified bassinet when sleeping, or cough and deep breathe every 2 hours, or IPPB without supervision every 4 hours.	2
	Continuous oxygen, or cough and deep breathe every hour, or continuous assisted ventilation.	4
	Mechanical respiratory aid, or IPPB with supervision every 4 hours.	8
	IPPB continuously with intermittent hand ventilation.	12
Suction	Routine postoperative standby.	1
	Nasopharyngeal or oral suction as needed	2
	Tracheostomy suction every hour, or nasogastric tube irrigation every 2 hours.	4
	Tracheostomy suction every 30 minutes, patient responsive.	8
	Tracheostomy suction every 30 minutes, patient not responsive.	12
Cleanliness	Bathes self, bed straightened.	1
	Bathes self with help or supervision, daily change of bed.	2
	Bathed and dressed by personnel, or partial bath given, daily change of linen.	4
	Bathed and dressed by personnel, special skin care, occupied bed.	8

Table II.3, cont'd.

Element	Description	Point Value
Turning and/ or assisted activity	Up in chair with assistance once in 8 hours.	1
	Up in chair with assistance twice in 8 hours, or walking with assistance	2
	Bedfast with assistance in turning every 2 hours, or up walking with assistance of two persons twice in 8 hours.	4
	Bedfast with assistance in turning every hour.	8
	Turning on orthopedic frame every hour.	12

Table II.4

PETO Conversion of Points into Hours of Care

Points	Physical Care Units (PCUs)	Hours of Nursing Care
4-11	1	1
12-19	2	2
20-27	3	3
28-35	4	4
36-43	5	5
44-51	6	6
52-59	7	7
60-67	8	8
68-75	9	9
76-83	10	10

Shuman [66]

Although the work of Shuman is primarily concerned with regional health system planning models designed to demonstrate methods of increasing productivity and minimizing the cost of providing services, it is worthy of further examination for two reasons. First, it provides insight into the model-building aspects of operations research as applied to health system problems. Secondly, an extension of Wolfe's model is presented which provides an answer to the allocation question not given in the original study.

The extension is made in the following way: with reference to (II.3) the variables x_{ij} are taken to be the number of times personnel type i performs task complex j. The x_{ij} are integers. The Y_i, on the other hand, are now assumed to be continuous variables. Finally, an additional constraint set

$$\sum_i x_{ij} = d_j \ , \qquad \forall j$$

is added to the model, where d_j represents the number of times task complex j is demanded. Obviously, the task complex demands may be assigned to several personnel types, and the resulting model solution yields an indication of personnel allocation. Shuman claims that by allowing Y_i to be continuous, a part-time staffing policy can be inferred from the model solution. This, in fact,

may be difficult to do because the optimal values of Y_i are as unlikely to terminate in half-time units as they are in full-time units. Thus, while the extended model is certainly an improvement over Wolfe's original model, it nevertheless is not completely representative of reality. Shuman provides a further modification to (II.3) indicating how this model may be used for several units simultaneously.

II.4 Patient Classification in a Long-Term Care Setting

Classification systems for long-term care have evolved essentially from those developed earlier for acute care. They have required some modification, mainly because of the need to specifically address the problems of aging and/or chronic disease. As will be seen, the studies have, in general, emphasized the following three areas:

1. Degree of self-sufficiency in daily activities;
2. Mental and emotional state;
3. Rehabilitative and instructional needs.

Katz, et al. [36]

The study of Katz, *et al.*, with respect to the recovery process of aged patients, is notable for its prescient recognition of the importance of degrees of patient independence in activities of daily living (ADL)

as they relate to nursing service required. The ADL's identified as being most important are

1. Bathing,
2. Dressing,
3. Toileting,
4. Transferring,
5. Continence,
6. Feeding,

with each further subdivided into a tri-level evaluation of independence. An eight category "Index of Independence in Activities of Daily Living" is offered, as shown in abbreviated form in Table II.5. This instrument was tested on patients representing numerous primary diagnoses. Katz found that almost universal applicability was possible regardless of diagnosis. Further, the phenomenon of increasing demand for nursing services as an ordered function of the index category was observed. The existence of factors of primary significance among the six in predicting degree of patient dependency was conjectured, but little follow-up study is reported.

McKnight [41]

As part of a study conducted to determine the amount and type of care received by nursing home patients in the Denver area, McKnight developed a set of "Criteria for Patient Nursing Need" which was used to specify a

Table II.5

Katz, et al. Classifications of Independence in ADL

Category	Description
A	Independent in feeding, continence, transferring, going to toilet, dressing, and bathing.
B	Independent in all but one of these functions.
C	Independent in all but bathing, and one additional function.
D	Independent in all but bathing, dressing, and one additional function.
E	Independent in all but bathing, dressing, going to toilet, and one additional function.
F	Independent in all but bathing, dressing, going to toilet, transferring, and one additional function.
G	Dependent in all six functions.
Other	Dependent in at least two functions, but not classifiable as C, D, E, or F.

three level (minimum care, moderate care, maximum care) patient classification system. These levels of classification are essentially analogous to the three levels of care Intermediate B, Intermediate A, and Skilled nursing care defined in Chapter I. The criteria identified were as follows:

1. Diagnostic description (primary and secondary);
2. Nursing activities (medications, treatments,

special diets, etc.);

3. Physiological factors (vision, hearing, continence, mobility, etc.);

4. Psycho-social factors (emotional and social behavior);

5. Rehabilitative factors (functional ability, both mental and physical).

Classification was dependent upon the similarity of patient condition with several descriptive items specified for each criterion and category.

A major portion of this study was devoted to determining the amount of care received by patients of the various classification levels. Time study analysis was undertaken to determine the performance times for 25 nursing care areas, both by patient classification and nursing skill level. In addition, activity sampling was performed in several of the nursing homes under study to give an indication of how nursing personnel time was allocated. It should be noted that the Denver work included the replication and extension of a study conducted by the City of Milwaukee Health Department [47].

Both the patient classification and nurse staffing aspects of McKnight's efforts will play a large role in our work. More will be said concerning this study in Chapter III.

Salmon, et al. [61,62]

The RAPIDS system developed by Salmon, *et al.* has as its major objectives

1. The forecasting of patient needs on input to the long-term care system, and
2. Placement assessment, both initial and follow-up.

A rating scale of one to five, one indicating no effort required and five indicating maximum effort required, was attached to each of the six areas denoted by the acronym

R - Restorative procedures
A - Activities of daily living
P - Problem behavior
I - Illness
D - Dependency, general
S - Social service

In applying this system, the authors discuss its successes in meeting the second of their objectives, but similar outcomes were not obtained for the first goal. No attempt was reported concerning the relationship between patient classification and staffing levels.

Burack [13], Burack and Denson [50]

Noting the need for an interdisciplinary set of indicators of geriatric patient status, Burack proposed a five-item list of guidelines as follows:

1. Medical classification (related to seriousness of disease);

2. Functioning performance (independence in ADL);
3. Projected goals (restoration of activity levels);
4. Therapeutic category (need for professional service, medications, or treatments);
5. Services required (medical, nursing, psychiatric, rehabilitative, social work).

Notice that the concept of goal-oriented care is emphasized in this system, a feature that has typically been relegated to minor status in other classification systems. Burack encourages further subdivision of the basic guidelines by care providers to better reflect their own needs. It is unfortunate that no indication of how this classification may be translated into nursing action is given.

Parker [52]

In the first of two studies designed to elicit a mathematically-based geriatric patient classification system, Parker examined the relationship between the 19 health status indicators and 4 health-care need status categories shown in Table II.6. Given that the binary vector $\underline{S}$ represents a patient profile in terms of the indicators and d_i, i=1,2,3,4, the health care status, it was demonstrated that Bayes' Theorem could be used to determine the probability of a patient with profile $\underline{S}$

Table II.6

Definitions of the Indicators and Categories

Used by Parker

Indicators (=1 if present, =0 if absent)

1.	Single (=1), Married (=0)	11.	Any social disability
2.	Drugs prescribed (=1 if at least one drug presceibed)	12.	Immobility--less than independent ambulation on level surface
3.	Cancer	13.	Needing to be fed
4.	Senility	14.	Special treatments needed
5.	Vascular lesions of the central nervous system	15.	Major assistance dressing and bathing
6.	Diseases of heart	16.	Conscious but unable to communicate needs
7.	Diseases of skin	17.	Aberrant behavior
8.	Arthritis	18.	Mild confusion
9.	Decubiti, ulcers, fistulas	19.	Severe confusion
10.	Total or occasional incontinence		

Health-Care Need Status Categories

1. Private home
2. Personal care, home
3. Skilled nursing home
4. Chronic disease hospital

being in category d_i from

$$P(d_i|\underline{S}) = P(\underline{S}|d_i)P(d_i) \div \sum_{i=1}^{4} P(\underline{S}|d_i)P(d_i) \qquad \text{(II.4)}$$

where

$$P(\underline{S}|d_i) \doteq P(S_1|d_i) \cdot P(S_2|S_1 \cap d_i) \cdot \ldots \cdot P(S_{19}|S_{18} \cap d_i) \quad \text{(III.5)}$$

and S_i indicates the presence of the i^{th} indicator. Using such a rule to classify 1,245 Medicare/Medicaid patients, Parker achieved a prediction rate of 70%. Methods were suggested for use in extracting a subset of the original variables which appeared most important in computing the categorization. These "key indicators" were found to be (in order of importance)

1. Mobility,
2. Continence,
3. Major assistance dressing and bathing,
4. Any special disability,
5. Severe confusion,
6. Special treatments needed.

Reducing the set of indicators by using only the key indicators above, a prediction rate of 70% was also achieved. Further comments on this approach will be made in Chapter IV.

<u>Parker and Boyd [53]</u>

In the second of his two studies, Parker explored both the use of and relationships between a discriminant

analysis approach and a multi-hierarchical cluster analysis approach to patient classification. Taking as data a sample of patient profiles derived from Collaborative Patient Assessment Instrument (CPAI)* scores on some 60 items, stepwise discriminant analysis was first applied in order to relate these items to one of six appropriate levels of care. An overall prediction rate of 77% was achieved. The clustering technique was then applied in order that patients with similar profiles might be grouped together. Five clusters were ultimately discovered, roughly analogous to the original six-level categorization. Taking cluster centroid vectors as being indicative of membership in that cluster, Parker proposed that a candidate's scores be added and compared to the totals for the five clusters as a rough classification rule.

Key variables identified by each method are shown in Table II.7. Those from stepwise discriminant analysis were determined by order of entry into the discriminant function, while cluster analysis variables were inferred from examination of how well each variable explained cluster membership (the "B/T" criterion).

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* A more complete discussion of this instrument and its development is given in Chapter III.

Further elaboration on both the CPAI and the statistical techniques employed will be provided in subsequent chapters, as well as analysis of the results. Note, however, the importance socio-demographic information plays in both findings, as well as the fact that no easily applied classification scheme was tested for its predictive capabilities. Neither of the results were related to staffing procedures.

Other

Two other classification studies are noted briefly. Ashton [3] developed a set of nursing criteria for evaluating extended care patients, the criteria being related ultimately to a tri-level (self-care, intermediate care, intensive care) classification system. Major factors identified were continence and mobility, mental and social status, independence in the ADL, and clinical nursing procedures required.

Shaughnessy, et al. [65] proposed a classification based on seven major factors (mental status, communication, sensory deprivation, physiological disturbances, physical dependence, environmental and psycho-social items) in order to assist in developing a triage categorization of patients. The categorization was then related to general personnel allocation guidelines.

Table II.7

Key Variables Determined by Discriminant and Cluster Analysis by Parker and Boyd

Discriminant Analysis	Cluster Analysis
1. Patient's present location	1. Communication of needs
2. Dressing	2. Present location
3. Length of time at location	3. Research site
4. Behavior pattern	4. Appropriate level of care
5. Age	5. Dressing
6. Stair climbing	6. Wheeling
7. Total family income	7. Toileting
8. Dentition	8. Mobility
9. Birthplace	9. Bathing
10. Toileting	10. Feeding
11. Mobility level	11. Bowel function
(Abbreviated list)	

II.5 Nurse Staffing in a Long-Term Care Setting

With one notable exception, staffing studies related to long-term care facilities have relied largely on the techniques of computer simulation to explore the questions of personnel mix, allocation, and assignment. We shall discuss three such simulation studies, and a mathematical programming approach.

For purposes of the discussions which follow, we note that computer simulation usually involves the modeling of a complex system by means of interrelated mathematical functions. Such an abstraction attempts to capture the essence of the manner in which various factors affect the system both directly and in combination with other variables. The model may be cast into either a descriptive or normative framework. Wagner [77] points out two basic caveats for such an approach:

1. If uncertainties are inherent in the model (e.g., the simulation of demand for service based on some probability distribution) results are subject to statistical error;
2. In order to make the system model computationally tractable, much abstraction and many simplifying assumptions may be required, with the undesirable result of divorcing the model from reality.

It seems clear that the authors cited below were aware of such difficulties.

Turner, et al. [74]

Noting the need in a nursing home environment for the determination of approximate resource levels (nursing, equipment, non-medical support, and outside medical assitance), and a reasonable scheduling system for such

resources as could be allocated, Turner, et al. proposed two simulation models with these goals in mind. These models incorporated a list of 28 direct care activities and 22 supporting (or indirect care) activities, and showed that about 80% of such activities could be scheduled, the remainder occurring more or less randomly. As stated by the authors, the models do not allow for parametric examination of the task performance time estimates used. Proposed measures of effectiveness by which various configurations were compared were the following:

1. Patient waiting time for service;
2. Specific task/skill level assignment patterns;
3. Amount of staff idle time;
4. The attainment of minimum service requirements.

McKnight and Steorts [43]

The model proposed by McKnight and Steorts represents a significant movement of simulation studies toward reality. Quite explicit descriptions of the nursing home environment in terms of number and size of units, patient mix (as per McKnight's classification system cited earlier), services offered, scheduling and occurence intervals of demands for service, staffing constraints, and task/skill level preferences are considered.

Beginning with the individual patient care plans translated into specific tasks and then aggregated within

classification levels, the model generates demands, both scheduled and random, with required service times taken from a normal curve fitted to task time-study data. To each demand is attached a priority, being a function of both patient classification and the approach of a cutoff time beyond which the demand would be unmet. Specific skill level assignments are made on the basis of a first and second choice preference structure.

In addition to generating financial reporting information, the model also reports the following:

1) Percentage of patient demands met, both with and without waiting;
2) Percentage of first and second choice personnel assignments;
3) Hours of care assigned by task/skill level; and
4) Staff utilization information.

Thus, from the manager's point of view, the relative effectiveness of various staffing configurations might be tested by making several model runs and comparing them on the basis of the above criteria.

Hundert [31]

In a very recent study, Hundert describes a two-phase nursing home simulation model whose dual objectives are the provision of the maximum amount of care at

minimum cost. The system takes as its input the following parameters:

1) Schedule of services--exact times or time intervals for task performance;
2) Server preferences--a first and second choice by task and patient classification;
3) Task priorities--set initially by the user, then updated in much the same way as in the McKnight and Steorts model;
4) Patient demand profile--a translation of the patient care plans in terms of the probability that a patient would require one of the specified services, with frequency and scheduling information also provided.

Through the use of time-study data, Hundert modeled service time distributions as gamma functions, which the simulation model can use to infer required task performance times.

Based on the output of this portion of the simulation, the cost phase of the model predicts average cost per patient day by means of linear regression estimates derived from historical data. Not surprisingly, it was found that nursing salaries, facility capacity and age, and patient census were the most important factors in the cost model.

The entire model was validated and used to determine the optimal unit size, staffing mix and levels, and service schedules for a proposed 180-bed nursing home. Acceptable results are reported for the first portion of the model, but cost estimates were deemed to be in question.

Liebman [38] and Liebman, et al. [39]

The work of Liebman represents a significant departure from earlier nurse staffing studies in the long-term care field in its reliance on the use of integer programming allocation and assignment models. In an attempt to develop "team effectiveness profiles" for comparison of various nursing team configurations in a hospital-based extended care unit (ECU), a short-term (or daily) assignment model and a long-term allocation model were proposed. Taking i as the index of individual patients, j the index of skill levels, and k the index of nursing tasks, the short-term model is formulated as

$$
\begin{aligned}
\text{MAX} \quad & \frac{1}{c^*} \sum_{i}\sum_{j} c_{jk} t_{ik} x_{ijk} \\
\text{s.t.} \quad & \sum_{j} x_{ijk} = 1, \qquad \forall\, i,k \\
& \sum_{i}\sum_{k} t_{ik} x_{ijk} \leq S_j, \qquad \forall\, j \qquad \text{(II.6)} \\
& x_{ijk} = 0,\ 1
\end{aligned}
$$

and the long-term model is

$$\begin{aligned}
&\text{MAX} \quad \frac{1}{c^{**}} \sum_{j}\sum_{k} c_{jk} y_{jk} \\
&\text{s.t.} \quad \sum_{j} y_{jk} = 1, && \forall\, k \\
&\qquad \sum_{k} T_k y_{jk} \leq S_j, && \forall\, j \qquad \text{(II.7)} \\
&\qquad\qquad y_{jk} \geq 0
\end{aligned}$$

The variables x_{ijk} in (II.6) indicate the assignment pattern of nursing skill levels to specific tasks and patients, where $x_{ijk}=1$ implies the assignment is made. In (II.7), the y_{jk} denote the fraction of demand for task k allocated to skill level j. The c_{jk} are skill level-task preference measures solicited from professional nurses by means of a Q-sort psychometric measurement technique. Although interval scaled measures were thereby obtained, equally acceptable results using corresponding ordinal relationships were reported. The t_{ik} and T_k are times required to perform specific tasks on an individual patient level or aggregated over all patients, respectively. These were determined by asking nurses how long each procedure usually took. Nursing time available to devote to direct care activities, S_j, was determined by aggregating task performance times for each skill level, again according to current assignment procedures. The constants c*

and c** are normalizing factors. Both models require that all tasks demanded must be accomplished. A combinatorial method relying on complete exhaustion of available time supplies was used to solve (II.6), while (II.7) was easily formulated as a network flow problem.

The validity of the model was tested by comparing the most effective assignment plans and team configurations computed with existing practices on the unit. Professional nurses were asked to state their preference between pairs of such results, with the model solutions usually being favored.

While this study represents a significant effort, it does leave room for improvement. First, its concepts are definitely hospital-oriented, albeit for an ECU, and the findings may therefore pose some difficulties for implementation in the typical long-term care facility. Task performance times and the method of computing the S_j tend to cause the solution to reflect heavily the status quo. Additionally, it is implicitly assumed that existing patient care plans may be readily formulated for use in the model in terms of the nursing task list presented. Finally, no provision is made for determining how constrained nursing resources can affect the amount of direct care given, and how choices related to task priorities for various patient types might be made and evaluated.

Mehta [44]

We briefly note a recent study by Mehta, which essentially quantified existing staffing and assignment procedures in sample facilities and categorized activities as direct care, indirect care, travel, communication, and personal time. By work sampling, the time devoted to each task was obtained and an indication of the skill levels performing each task was given. The staff mix was determined by simply dividing the aggregate time spent by each skill level by seven hours to obtain full-time equivalent personnel. The use of a float staff was recommended to handle time for which no accounting could be made by the above procedure. Not surprisingly, the staffing recommendations that were provided closely resemble the staff mix that existed on the units at the time of the study.

II.6 Work of the "Four University" Group

We conclude our survey by noting the contributions made in several areas of the long-term care field by the so-called "four university" group. Established in the late 1960s, the study team consisted of those researchers listed in Table II.8 with primary research interests as indicated. Recognizing the many schemes for long-term care patient classification already in existence, the

Table II.8

The "Four University" Research Group

Group Location	Principal Investigator	Purpose of Primary Interest
Case-Western Reserve Dept. Preven. Med.	Dr. S. Katz* Dr. M. Stroud**	Studies of course of illness in chronic disease
Harvard Center for Community Health and Medical Care	Dr. Paul Donsen Mrs. Ellen Jones	Evaluation of programs for care of the chronically ill
Hospital Assn. of New York State	Mr. L. Danehy	Community resource allocation
Johns Hopkins Sch. of Hygiene & Pub. Hlth. Operations Research Division	Dr. C. Flagle	Facilitating decision making in management of patients

* Now at Michigan State University
** Now at University of Pennsylvania

group saw its main task as the development of an assessment system that would not only capture the important aspects of then current systems, but also establish a "common language" of patient classification that would be interdisciplinary in scope. The result was the Collaborative Patient Assessment Instrument (CPAI) alluded to earlier, along with its accompanying user's manual [35]. A detailed discussion of the CPAI will be undertaken in

the next chapter. For the present, it is sufficient to call attention to the statement of Jones [34], which points out the significance of the CPAI as a patient-oriented rather than process-or service-oriented system.

Although the original group has officially disbanded, the individual research teams have continued to pursue their interests in the LTC field in a collaborative manner. Among those studies currently underway are the improvement of the assessment tool as a means for improving the quality of patient assessment, and additional work on the relationships between patient classification, demands for care, and facility nurse staffing.

II.7 Summary

This chapter has attempted to describe briefly some relevant studies concerning patient classification and nurse staffing in both the acute and long-term care environments. Commonalities in conceptual approaches, significant improvements, and areas for further exploration have also been highlighted. It is against this background that the models and procedures proposed in this dissertation are presented. It is hoped that the historical perspective that has been provided will prove helpful in evaluating the new methodologies.

Chapter III. Development of the Methodology

III.1 Introduction

Having discussed the history and current state of the art of nurse staffing methodologies we turn now to the development of the proposed models. Recall that we essentially are seeking a process that will assist a LTC facility administrator in solving personnel budgeting problems by presenting alternative courses of action for consideration. Unlike some previously designed systems of this sort, however, the approach suggested here relies heavily on the concept of patient needs as related to demand for nursing service. As a result, supplementary information is concurrently produced that will benefit the director of nursing in allocating the predicted nursing resources.

After an initial discussion of the general problem to be solved, we will demonstrate how patient needs may be related to nursing activity through classification. The insights obtained will lead directly to the construction of a mathematical programming model and two modified formulations derived from it. These models will serve as the basis for solving the budgeting and nurse staffing problems.

III.2 The Administrator's Problem

As was pointed out in Chapter I, the LTC facility administrator, as the primary managerial decision-maker, is responsible for the total allocation of available resources so that the essentials of institutional life may be provided. Given that the bulk of operating expenses are attributable to personnel costs, especially those of nursing staff, we chose to concentrate primarily on the manner in which such expenses could be related to the provision of service. Recall, too, that the five considerations identified by Aydelotte [5] (quality of care, patient characteristics, staffing predictions, staffing logistics, and quality measurement) were postulated as being of consequence in any budgeting process.* Additionally, certain exogenous variables must be considered by the administrator in reaching an acceptable decision as to personnel budget and the staffing mix it would imply. These variables include the following:

1. State and federal requirements for minimum staffing and skill level coverage (e.g., R.N. on duty at least one shift per day, adherence to a staff-to-patient ratio of 1:25, etc.);
2. Reimbursement rates for Medicare/Medicaid patients;
3. Demand intensity implied by the facility's

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*These items were discussed in Chapter I.

patient mix.

The synthesis of all of the above-mentioned factors into a normative, decision-making framework, along with such intangibles as the desire to maintain a good professional reputation, is part of what Baloff, Abernathy, and Hershey [8] identify as the "aggregate budgeting process." The decision variables involved in this process are the budget to be allocated to nurse staffing and the staffing levels and skill mix to be maintained. Essentially, then, the administrator's problem may be said to be

> How to fix budget and staffing levels in a manner responsive to patient needs and other factors, both for the short run and as a result of future alterations in the exogenous and/or endogenous considerations mentioned earlier (e.g., legal requirements, reimbursement rates, patient mix, etc.).

Baloff, _et al_. [8, p.4] succinctly capture the essence of why this problem is non-trivial:

> ...The difficulties of nurse staffing planning are partially caused by two insurmountable problems for which no complete remedies exist: (1) the impossibility of developing perfect demand forecasts for the future, and (2) the inability of specifying exactly and unequivocally the manner in which demand should be translated into required staff.

To these points we might also add the inherent pitfalls in the intermediate step of relating patient health status to demand.

Recognizing the major areas of difficulty in nurse staffing planning, as noted above, it is perhaps wise to view the administrator's problem in terms of the resolution of these problems. Certain of the exogenous changes to which a response is desired may be captured in the model which translates demand into required staff. The relationship between patient health status and demand, along with the development of future demand forecasts, may be viewed as a separate but interrelated entity. While we do not propose to predict future patient status in a manner analogous to Singer [68], we may rely on the relative stability of the patient mix within LTC facilities to lessen the need for such a procedure. Then, too, a method of assessing patient health status which may be readily applied at frequent intervals decreases the necessity for long-range demand forecasting, since it is usually the case that such predictions are required in the absence of better information. With this overall solution framework in mind, we turn now to the development of a demand-related patient classification system.

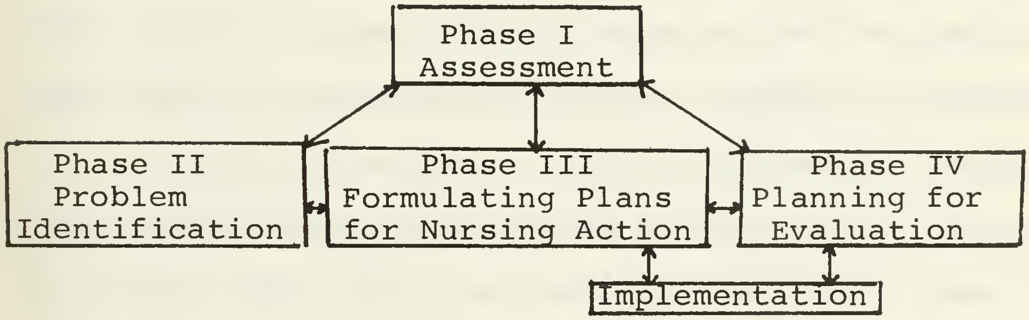
III.3 A Patient Classification System

The lack of a truly useful patient classification methodology for our purposes in the long-term care setting has been previously noted. Such systems as do exist fall short of the goal of relating patient health status to demand for nursing service for essentially two reasons: (1) health status is, in fact, not adequately reflected, and (2) demand predictions are overly simplified and not readily applicable to the aggregate budgeting process. By way of example of the first shortcoming, we cite the use of patient "level of care" placement. Since the placement decision must, of necessity, include the consideration of several other factors in addition to patient health status, these factors may at times be so important as to alter any decision based solely on health status. Then, too, the time lags involved in the reassignment of a patient whose health status has changed can cause the current placement to be a poor predictor of nursing needs. The second shortcoming is exemplified by the PETO system, where the predicted demands for nursing services are stated in terms which do not allow for determination of the skill level mix as it relates to budgeting. It should be noted that other methods, such as those proposed by Burack [13,50] and Katz [36] were never shown to be related to demand for nursing service.

Perhaps the ideal vehicle for reaching our stated goal is the intermediate step of a patient care plan. Having evolved from the traditional nursing care plan by including patient data of interdisciplinary interest, the patient care plan in its most recent form is usually presented in a "problem-oriented" format, and relies on a thorough consideration of patient status. Such a structure requires recording of both subjective and objective problem identification and assessment, a plan of approach, and the specific actions required to ameliorate the difficulties. Often included are expected goals to be attained as a result of treatment. The implementation of this approach at the nursing level is shown schematically by Bower [10, p.10] in Figure III.1.

Figure III.1

Bower's Process of Planning Nursing Care



It is of note that this form of patient care plan incorporates a one-to-one correspondence between patient status and nursing demands. Aggregated over all patients in a facility, these demands would provide a reasonable indication of required staffing. There are, however, three drawbacks to this approach: (1) skill level preferences for task assignments are often omitted, (2) examination of alternative budget levels and skill level mixes would be difficult, and (3) the complex relationship between the myriad of possible nursing actions and performance times would be difficult to identify and standardize. On a more mundane level, the problem-oriented patient care plan has yet to be fully implemented in a significant number of long-term care facilities. Indirect use of care plans through translation of indicated activities into tasks listed on a predetermined schedule has been suggested by Liebman [38,39] and in the several simulated models cited in Chapter II. Such procedures, however, can be extremely time-consuming, and can require a great deal of professional nursing judgment to implement.

Given the state of the art in patient classification systems and the inherent difficulties in the use of patient care plans for budgeting purposes, there is a need for a classification system emphasizing ease of application and comprehensive assessment and demand

prediction. That is, the categorization of patients into distinct, homogeneous groups should take place with the following criteria in mind:

1. The classification decision should have its roots in a comprehensive assessment of a patient with respect to functioning status, impairments, medical status, psycho-social status, and medically defined conditions;
2. The classification groupings must bear some well-defined relationship to demand for nursing services.

In keeping with the first of these guidelines, the proposed classification system is to be based on the assessment information contained in the Collaborative Patient Assessment Instrument (CPAI) developed by the "four university" group.* A listing of the classification descriptors is given in Table III.1, and an illustration of the instrument itself is shown in Appendix A. As can be seen from the descriptors included, the overall theme of the assessment is problem-oriented instead of diagnostically dependent. Jones [35] further identifies the following four characteristics of the descriptors:

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* The organization of this group was discussed in Chapter II.

Table III.1

CPAI Classification Descriptors

Identifying and sociodemographic items:
- Date of classification
- Interviewer or Classifier
- Program or project identification
- Patient's record or study number
- Social security number or health insurance benefit number
- Birth date
- Birthplace
- Sex
- Race
- Marital status
- Religious preference
- Residence address
- Patient's location at time of assessment
- Length of time at location
- Usual living arrangements
- Number of living children
- Education
- Usual occupation
- Employment status
- Total family income
- Health care coverage

Functioning status items:
- Mobility level
- Transferring
- Walking
- Wheeling
- Stair climbing
- Bathing
- Dressing
- Eating/feeding
- Toileting
- Bowel function
- Bladder function
- Communication of needs
- Orientation as to time, place, person
- Behavior pattern

Impairment items:
- Sight impairment
- Hearing impairment
- Speech impairment
- Fractures and dislocations
- Joint motion disorders
- Joint pain and swelling
- Missing limbs
- Paralysis/paresis
- Dentition

Medical status: risk factor measurements:
- Height
- Weight
- Blood pressure
- Blood cholesterol
- Kidney function measurement: BUN
- Albuminuria
- Cigarette smoking

Certain medically defined conditions:*
- Alcoholism
- Anemia
- Angina and/or myocardial infarction
- Arthritis
- Cardiac arrhythmias
- Congestive heart failure
- Diabetes mellitus
- Drug abuse
- Hypertension
- Malignancy
- Mental illness
- Neurologic disorders
- Respiratory disease, chronic

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* In addition, all medical diagnoses may be listed.

1. Patient orientation: the individual is described as he or she actually is, not in terms of services being rendered or physical location;
2. Multidimensionality: The scope of the assessment is a broad one;
3. Objectivity: little subjective or interpretive analysis is required, hence results are reproducible;
4. Relevancy: the items included have been shown to be strongly related to the major issues of morbidity and mortality.

In addition, the subcategorizations provided with the functioning status, impairment, and medically defined items serve to indicate the patient's degree of dependency with respect to each descriptor. The incorporation of the suggested coding provided in the CPAI User's Manual [35] thus yields an ordinally scaled set of variables for purposes of analysis.

As part of their developmental work on the CPAI, three of the research groups undertook the field testing of the instrument in various locations. As a result, a total of 623 assessments on Medicare/Medicaid patients was obtained, the sample consisting of those patients whose placement levels were as shown in Table III.2. As

Table III.2

CPAI Sample Categorized by Level of Care and Research Group

Level of Care	Research Group HANYS	Johns Hopkins	Harvard	Total
Private Residence	---	47	---	47
Rented Room(s)	2	---	---	2
Domiciliary/Personal Care Facility	---	18	---	18
ECF/Supportive Nursing	30	70	---	100
Nursing Home	63	53	---	116
ECF/ECU	31	3	---	34
Chronic Disease/ Rehabilitation Hospital	---	11	295	306
Total	126	202	295	623

an additional and highly important part of this study, criteria for judging a patient's most appropriate level of care were established and utilized by various health professionals to obtain an indication of the divergence between legal placement and placement based on judgment of patient needs. The resulting categorization, which illustrates the observed divergence, is shown in Table III.3.

The appropriate placement decision was made, for the most part, on the basis of the patient's health and

Table III.3

CPAI Sample Categorized by
Appropriate Level of Care and Research Group

Appropriate Level of Care	HANYS	Johns Hopkins	Harvard	Total
Private Residence/Room/ Home with or without Home Care	1	59	4	64
Personal Care Facility	---	15	5	19
ICF/Supportive Nursing Care Facility	34	77	54	165
Skilled Nursing Home (ECF/ECU)	91	45	68	205
Chronic Disease/ Rehabilitation Hospital	---	6	157	163
Other Hospital	---	---	7	7
Total	126	202	295	623

mental status. Where there were some overwhelmingly important factors among the patient's sociodemographic characteristics (e.g. relatives living at home, insurance arrangements, etc.), the placement decision may have been altered. Similarly, borderline placement questions were usually resolved by recourse to this information. By and large, however, the adjudged appropriate level of care provides the high degree of association with patient status (obtained through assessment) that we seek in

keeping with the first criterion for classification.

Quantification of the relationship between this judgmental variable and the multivariate indication of health status provided by the CPAI can therefore become the basis for an operable patient classification system. Due to the nature of the data involved, and in keeping with the need for a readily-applied methodology, a regression-based multivariate statistical approach has been utilized. Further explanation of specific methods, rationale for the approach, detailed description of the application of the method of choice, and the resulting classification system are provided in Chapter IV.

There remains the consideration of the link between the derived classification system and nursing service demands. When others have attempted to quantify these demands it has typically been the case that the time necessary to perform the several tasks identified with the functions of nursing are presented. These performance times are usually, although not necessarily, obtained from time-study experiments, some typical examples being given in Liebman [38,39], McKnight and Steorts [43], and Hundert [31]. Unfortunately, the task listings usually contain anywhere from 65 to 75 items which, when combined with the various patient classifications and nursing skill levels in a staffing model, can

lead to models too large for most computers to solve.

In a nursing home research study conducted in Denver, Colorado, however, McKnight [41] proposed the grouping of tasks typically performed in long-term care facilities into 25 "care areas" as shown in Table III.4. Notice that in addition to tasks which have traditionally been identified as "direct care" items (i.e., performed in the presence of the patient), there is also involved an area identified as "associated nursing care functions." The constituent tasks of this grouping, such as charting and rounds, are certainly more patient-oriented than other "indirect care" tasks performed by nurses. For this reason McKnight [42] identifies the 25 care areas as "patient centered" instead of as direct care groupings.

A major portion of the Colorado work was devoted to time studies of nursing activities in 14 homes generally judged as giving "good" quality care. Specifically, the amount of time devoted by nursing personnel to the constituent tasks of the 25 care areas over a 3 1/2-day period for 195 sample patients was obtained. The findings shown in Table III.5 indicate the average time, <u>per occurrence</u>, for each care area by patient classification (minimum care, moderate care, maximum care). Although this is a common way of presenting this type of result, such data are difficult to apply without prior knowledge

Table III.4
Nursing Care Areas Identified by McKnight

01. ELIMINATION
   1. Toilet
   2. Commode
   3. Bedpan
   4. Urinal

02. CATHETERIZATION (and related procedures)
   1. Catheterization
   2. Insertion of Foley catheter
   3. Bladder irrigation
   4. Emptying drainage bottle

03. DAILY CARE
   1. Morning care
   2. Evening care
   3. Rest period
   4. Night check

04. BATHING
   1. Complete bed bath
   2. Partial bed bath
   3. Partial bath
   4. Shower
   5. Complete tub bath

05. BED MAKING
   1. Occupied
   2. Unoccupied
   3. Straightening beds

06. CARE OF SKIN
   1. Preventive skin care
   2. Treatment of decubitus
   3. Gentle massage

07. CARE OF HAIR AND FACE
   1. Combing and braiding
   2. Shampooing
   3. Cutting
   4. Setting hair
   5. Scalp treatment
   6. Shaving

08. ORAL HYGIENE
   1. Brushing teeth
   2. Denture care
   3. Mouth care
   4. Care of lips

09. CARE OF NAILS
   1. Fingernails
   2. Toenails
   3. Manicure or pedicure

10. REHABILITATION and RECREATIONAL
   1. Passive exercise
   2. Supervised exercise
   3. Diversional

11. FOOD and NOURISHMENT
   1. Breakfast
   2. Lunch
   3. Dinner
   4. Extra feedings
   5. Water

12. MEDICATIONS
   1. Oral
   2. Intramuscular
   3. Subcutaneous
   4. Instillation of drops
   5. Intravenous
   6. Rectal
   7. Inunction

13. CARDINAL SYMPTOMS
   1. Mouth temperature
   2. Rectal temperature
   3. Radial pulse
   4. Blood pressure

14. THERAPEUTIC MEASURES
   1. Hot water bottle
   2. Ice bag
   3. Compress
   4. Hot soak
   5. Heat lamp
   6. Ultra-Sound
   7. Diathermy
   8. Whirlpool
   9. Bandages

15. URINE TESTS
   1. Clinitest
   2. Acetone test
   3. Testape or uristix

16. ENEMA
   1. Cleansing
   2. Retention
   3. Removal of fecal impaction
   4. Insertion of colon tube

17. CARE OF COLOSTOMY or ILELOSTOMY
   1. Change of dressing
   2. Irrigate in bed
   3. Irrigate in toilet room

18. ASSOCIATED NURSING CARE FUNCTION
   1. Telephone contact or request
   2. Personal contact
   3. Assistance with procedures
   4. Charting

19. WEIGHING PATIENT
   1. At bedside
   2. In weighing room

20. GAVAGING PATIENT

21. FEMALE PROCEDURES
   1. Vaginal douche
   2. Perineal care

22. OXYGEN THERAPY
   1. Cannula
   2. Catheter
   3. Mask
   4. Tent

23. CARE OF CRITICALLY ILL
   1. Acute condition
   2. Comatose condition
   3. Dying condition

24. PREPARING BODY AFTER DEATH

25. MISCELLANEOUS

Table III.5
Mean Time* Needed to Complete Each of the Nursing Care Areas by Personnel Level and by Patient Ambulatory Status and Nursing Need as Found by McKnight

AREA	OVERALL MEAN	PERSONNEL LEVEL				AMBULATORY STATUS		NURSING NEED		
		RN	LPN	Aide	Other	Amb	Non-Amb	Minimum	Moderate	Maximum
Elimination	3.90	3.27	3.44	3.91	4.88	3.46	4.32	3.49	3.44	5.38
Catheterization	3.75	8.19	4.86	3.23	0.00	8.75	3.50	2.00	3.86	3.72
Daily Care	3.37	0.97	2.21	3.70	8.64	2.64	4.07	1.07	3.71	3.96
Bathing	15.67	0.00	12.00	15.64	20.13	15.84	15.50	15.32	15.79	15.61
Bed Making	4.91	2.75	7.68	4.87	4.64	4.61	5.34	4.73	4.86	5.33
Care of Skin	5.32	6.25	5.44	5.28	0.00	4.58	5.38	11.50	5.76	4.96
Care of Hair & Face	3.28	0.00	0.46	2.97	7.88	3.74	2.89	4.41	3.49	2.52
Oral Hygiene	2.23	1.25	5.17	2.14	2.56	1.93	2.49	0.88	2.23	2.34
Care of Nails	4.45	4.25	2.13	4.38	6.75	4.35	4.62	3.71	4.69	4.46
Rehabilitation & Recreation	5.43	1.96	2.37	3.25	10.25	4.84	6.23	4.55	6.12	4.41
Food & Nourishment	2.29	1.16	1.91	2.45	1.23	1.40	3.65	1.27	1.78	5.30
Medications	1.81	1.79	1.89	1.51	4.75	1.75	1.90	1.47	1.89	2.13
Cardinal Symptoms	2.35	1.34	1.65	3.19	1.58	2.11	2.58	2.01	2.23	2.66
Therapeutic Measures	5.77	9.10	4.67	4.83	7.92	5.54	5.96	2.50	6.15	6.05
Urine Tests	2.87	1.80	2.38	3.02	0.00	2.87	2.86	3.03	2.79	3.88
Enema	8.20	0.00	7.38	8.42	0.00	8.71	7.96	0.00	9.00	7.31
Care of Colostomy or Ileostomy	6.57	0.00	0.00	6.57	0.00	6.57	0.00	0.00	6.57	0.00
Associated Nursing Care Function	0.63	0.57	0.61	0.67	2.52	0.62	0.65	0.58	0.65	0.67
Weighing Patient	1.29	0.00	0.00	1.29	0.00	1.29	0.00	0.95	1.54	0.00
Gavaging Patient	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Female Procedures	1.83	0.00	0.00	1.83	0.00	0.00	1.83	0.00	1.83	0.00
Oxygen Therapy	2.75	0.25	0.75	3.65	0.00	0.75	3.08	0.00	0.75	3.08
Care of Critically Ill	3.25	4.00	0.00	2.50	0.00	0.00	3.25	0.00	3.25	0.00
Preparing Body after Death	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Miscellaneous	1.06	0.82	0.69	1.14	1.22	0.98	1.20	1.01	1.02	1.28

* Includes only single person activities.

of the existing individual patient care plan; that is, an indication of the potential number of occurrences of a care area for each patient. Since our aim here is to relate demand to homogeneous patient categories, performance times for a "typical" or "average" patient of each group would be the more appropriate measures. In order to derive such information, the source data for the Colorado study were obtained, and aggregate times by care area for all patients found. These times include an allowance for the procedural steps of each activity within a care area that were not performed but were determined by professional judgment to be necessary. The use of time information in this form is appropriate to the normative models to be proposed. At this point, the aggregates were divided by the number of patients in each category, and then divided again by 3 1/2 days. The resulting information is thus interpreted as the average required performance times to be devoted to each care area for an "average" patient of each category per 24-hour day. Note that we are implicitly assuming that the pattern of demands as demonstrated in the sample data is a reasonable representation of the pattern that would exist in populations of similarly classified patients. Completely random selection of the patient sample for the original study lends credence to this assumption. The

average required care area performance times by patient classification are given in Table III.6. As a matter of interest, the average total nursing time per patient day obtained by summing the care area performance times is .61 hours for minimum care patients, 1.55 hours for moderate care patients, and 2.77 hours for maximum care patients.

Recall that in Chapter II we noted that McKnight's tri-level classification system may be interpreted as being analogous to the three levels of care denoted as Intermediate B, Intermediate A, and Skilled Nursing care. For the appropriate level of care used in our classification system the analogous levels as shown in Figure III.3 are Personal Care Facility, ICF/Supportive Nursing Facility and Skilled Nursing Home (ECF/ECU). The result we seek in keeping with the second of our classification system criteria is thus immediate. The implicit demand for patient-centered nursing services based on patient health status can be inferred by first classifying the patient into one of the three categories (Int. A, Int. B, Skilled), and then utilizing the average required performance time information available from Table III.6 for the corresponding classification. Since this time/demand data is for an "average" patient, the result is most powerful when used to determine demands by care area and patient classification for the residents of a large unit

Table III.6

Average Required Performance Times Per Day (in Minutes)
by Care Area and Patient Classification
(Based on McKnight's Colorado Study Data)

Care Area	Patient Classification Minimum (N=64)	Moderate (N=95)	Maximum (N=36)
1. Elimination	.53	4.94	8.24
2. Catheterization	.02	1.00	5.84
3. Daily Care	2.36	23.23	43.79
4. Bathing	2.59	8.09	13.84
5. Bed Making	3.46	6.11	6.41
6. Care of Skin	.05	1.79	11.10
7. Care of Hair and Face	.75	2.89	3.02
8. Oral Hygiene	.02	.58	1.20
9. Care of Nails	.13	.35	.35
10. Rehabilitation and Recreation	1.31	4.61	6.68
11. Food and Nourishment	8.56	17.35	36.32
12. Medications	5.99	9.27	10.45
13. Cardinal Symptoms	.61	.90	2.90
14. Therapeutic Measures	.04	.44	1.92
15. Urine Tests	.17	.49	.18
16. Enema	.12	1.00	1.00
17. Care of Colostomy or Ileostomy	*	.21	*
18. Associated Nursing Care Function	4.13	6.17	6.21
19. Weighing Patient	.04	.04	*
20. Gavaging Patient	*	*	*
21. Female Procedures	*	.02	*
22. Oxygen Therapy	.12	.09	.16
23. Care of Critically Ill	.20	.11	*
24. Preparing Body After Death	*	*	*
25. Miscellaneous	5.05	5.50	6.57

* Not observed.

or an entire facility. The rationale is that the averaging process used in the derivation of required performance times would be replicated if a large enough patient population were considered at the facility level.

The need for a demand-related classification system has thus been met. Note that since we are considering a facility oriented problem the "home care" classification is not considered. On the other hand, neither do we consider the "chronic care" category, mainly because of the lack of supporting time data. Nevertheless, the proposed system is well suited for the majority of long-term care facilities and is in consonence with the trend toward multi-level certification. In the next section we shall demonstrate how the demands derived by this method may be allocated among available nursing personnel.

III.4 A Basic Staffing Model

Given that we now have a means whereby patient needs may be translated into demand for patient-centered nursing activities through the intermediate steps of classification and aggregation, we now examine how this demand is related to staffing. This issue is the heart of the facility administrator's problem. Recall that the basic decision variables under the administrator's

control are the budget to be allocated to nursing staff and the number and skill level mix of nurses to be maintained. An additional decision to be reached is how the available nursing resources may be assigned to various nursing functions. This latter concern is usually within the purview of the Director of Nursing.

Decisions must be made, however, in an environment constrained by several factors. Alluded to earlier, these include the following:

1. An upper limit on funds which may be devoted to staffing;
2. Legal staffing minimums and guidelines;
3. Satisfaction of patient needs through nursing intervention;
4. Limitations on the amount of nursing time available to devote to patient care due to other staff responsibilities;
5. Assurance that patients with high priority nursing needs are not neglected.

With all such restrictions in mind, decision variables may be fixed at those levels which act to optimize whatever criterion is considered most important. If dollar cost of nurse staffing is chosen, for example, it is submitted that a minimum cost solution would not be difficult to find by using legal constraints as a basic

quideline and heuristic procedures thereafter to meet as many other restrictions as desired. We propose, however, a normative approach using as a criterion the appropriateness of assignment of nursing personnel as a surrogate measure of quality of care.

The techniques of mathematical programming are well-suited to model building of this type; that is, the optimization of a criterion function subject to certain constraints, both of which may be expressed mathematically. Taking personnel budget as a parameter and staff numbers, skill level mix, and assignments as variables, the postulated model is cast into the mixed-integer linear programming form

$$\begin{aligned} &\text{MAX } \underline{cx} \\ &\text{s.t. } A\underline{x} = \underline{b} \\ &\qquad x_j \geq 0 \text{ and integer}, \quad \forall j \in N_1 \qquad \text{(III.1)} \\ &\qquad x_j \geq 0, \quad \forall j \in N_2 \end{aligned}$$

Specifically, the Basic Staffing Model (BSM) is given as

$$\text{MAX} \sum_i\sum_j\sum_k c_{ijk}x_{ijk} + \sum_j\sum_k p_{jk}\beta_{jk} \qquad \text{(III.2)}$$

$$\text{s.t.} \sum_i s_i n_i \leq \text{Budget} \qquad \text{(III.3)}$$

$$\sum_j\sum_k x_{ijk} - a_i n_i \leq 0, \quad \forall i \qquad \text{(III.4)}$$

$$\sum_i x_{ijk} - \beta_{jk} = 0, \quad \forall j,k \qquad \text{(III.5)}$$

$$L_i \leq n_i \leq U_i \text{ and integer}, \quad \forall i \qquad \text{(III.6)}$$

$$x_{ijk} \geq 0, \quad \forall i,j,k \qquad \text{(III.7)}$$

$$L_{jk} \leq \beta_{jk} \leq U_{jk}, \quad \forall j,k \qquad \text{(III.8)}$$

where

$i=1,2,\ldots,I$ indexes the various nursing personnel skill levels, $j=1,2,\ldots,J$ indexes the care areas, and $k=1,2,\ldots,K$ indexes the levels of patient classification. Further, the following quantities are defined:

c_{ijk} = a constant measuring the appropriateness per unit time of having skill level i assigned to care area j for patient class k. $c_{ijk} \geq 0 \ \forall i,j,k$.

p_{jk} = a dichotomous (0,1) constant, with a value of 1 denoting the effectiveness per unit time of a care area/patient classification combination deemed to have a high relative priority, and a value of 0 denoting a low priority combination.

x_{ijk} = a variable which indicates the amount of time devoted by skill level i to care area j and patient class k in a model solution.

s_i = a constant representing the dollar cost per unit time for skill level i.

n_i = a variable delineating the number of members of skill level i to be included in the staff mix.

a_i = a constant indicating the number of time units available for assignment to patient centered activities for skill level i (i.e., total available time less time needed for other nursing and personal activities).

β_{jk} = a variable specifying the amount of time allocated by the model solution to care area j for patient class k.

L_i, U_i = upper and lower restriction constants on the number of personnel of skill level i which may be included in the model.

L_{jk}, U_{jk} = upper and lower restriction constants on the amount of time to be allocated to care area j for patient class k.

With this information available, we examine the constituent parts of the Basic Staffing Model to illustrate how each relates to the administrator's problem.

The objective function (III.2) indicates the maximization of the sum of both a time-related appropriateness measure over all possible skill level/care area/ classification combinations and a time-related priority measure. The meld of skill levels with jobs produced serves as a surrogate measure of quality of care in keeping with the findings of Revans [57,58]. More will be said on the priority aspect below. In addition, Liebman [38,39] demonstrated that an objective function given in terms of performance times produced superior assignment patterns to objective functions without such a feature when used in models of this type.

Constraint (III.3) simply assures that the upper limit on personnel budget will not be exceeded. In order to insure against assigning more time than is available for any skill level, the constraints (III.4) restrict all possible assignments of skill level i to be less than or equal to the time available for patient-centered activities.

The constraint sets (III.5) and (III.8) when considered together serve a dual purpose. The first of these is to assure that when the assigned times of all skill levels to care areas and patient classifications are added, patient demands for all combinations are satisfied. Note, however, that the demands themselves are model

variables, albeit with certain minimum times which must be met and maximum times that cannot be exceeded. The maxima and minima for every care area/classification/demand combination serve as a starting point for the consideration of priority of patient needs.

That some activities are given priority over others in the resource constrained environment of a long-term care facility is a well-known fact. Little and Carnevali [40, p. 53] suggest that attention should be given to, "...those problems which are most relevant to the patient's well being, keeping in mind the realities of staffing levels and the abilities of those who contribute to patient care." They go on to suggest the existence of a hierarchy of patient needs within which physiological problems would be considered first, followed by safety, love, esteem, and self-actualization. Bower [10], on the other hand, postulates a hierarchy of needs wherein problems threatening a patient's life, dignity, or integrity are considered first; with problems which may destructively change the individual considered next; and those affecting the developmental growth of the individual following. It is emphasized, however, that priority setting should not negate the importance of those demands considered to be of lower priority, but rather to provide a means by which demands may be

satisfied efficiently.

With these ideas in mind, we can incorporate a rudimentary priority system into the Basic Staffing Model in the following manner. Assume that we can dichotomize the set of care area/patient classification combinations into a set containing those combinations of relatively high priority and a set with combinations of lower priority. Guided by the average performance times as shown in Table III.5, the upper and lower bounds on β_{jk} are set in a manner indicative of the priority class for (j,k). That is, relatively high priority combinations might have their lower bounds set at or above the average time, and the upper bound set at some greater value. Lower priority items, on the other hand, might have their lower bounds set below the average time, and the upper bound at or near the average. Such a framework conceptually insures that, in a situation of constrained resources, high priority combinations will receive an adequate allocation of nursing time, while lower priority combinations will be allocated at least their lower bound.

Given the dichotomy of care area/patient classification combinations and the loosely defined priority system it implies, a modification to the objective function as shown in (III.2) is possible. That is, we would like to insure that should additional nursing resources

be available after satisfaction of the lower bounds, they would be assigned to the higher priority combinations. Thus, if we set $p_{jk} = 1$ for the high priority combinations, the objective function would include a measure reflecting the importance of assigning additional time as is available to high priority combinations. In our rudimentary system, note that no further priority ranking is given. Such a determination might be the subject of future research. The entire process described above is obviously highly subjective, as is the entire concept of priority setting. It is, however, believed to be a reasonable way to handle a difficult problem using the power of the mathematical model.

Finally, constraint set (III.6) allows the specification of staffing level restrictions, again by using upper and lower bounds. Minimum coverage by skill level may be obtained by using the appropriate lower bounds. Staff-to-patient ratios may also be incorporated here. The upper bounds can be set to reflect the exigencies of the hiring prospects for a skill level, or simply to restrict the allowable number of nurses of each skill level the model may assign. Note that the variables n_i have been restricted to be integers. By computing the availabilities a_i on the basis of an eight-hour shift, the integrality restrictions imply the assumption

of no part-time personnel. Although this is the manner in which we propose to use the model, an extension to a part-time personnel policy is feasible. For example, if the a_i were computed for a four-hour shift, the number of half-time personnel would be obtained, it then being an easy step to determine the full- and part-time staffing indicated.

An additional and very important aspect of the Basic Staffing Model is its adaptability for use in a variety of circumstances. Parameters may be set in such a way that staffing and assignment patterns for any given unit and/or shift may be calculated, assuming the appropriate information is available. Its greatest potential, however, is realized in aiding the solution of the aggregate budgeting problem. In fact, due to the nature of the available performance time data, it is best applied using a 24-hour day as the basic time frame. Taking the parameters a_i to reflect per shift availability, total daily staffing requirements and gross assignments of skill levels to care areas and patient classes can be derived. Information presented in this fashion allows for the flexible scheduling of the daily routine according to shift and individual personnel by the director of nursing in any way deemed appropriate for a particular facility.

III.5 Extensions to the Basic Staffing Model

Assuming that the Basic-Staffing Model is an accurate normative form of the administrator's problem, an optimal solution will yield sufficient information to enable specification of the best feasible staffing mix; that is, subject to the assumed constraints on budget, patient mix, and so on. Although this optimal solution is important of itself, however, the model from which it was obtained allows of no information concerning alternative courses of action. That such information is of consequence to the administrator should be quite clear. The potential impact on staffing of the internally and externally controlled factors alluded to in our discussion of the administrator's problem must be weighed in reaching an acceptable solution. Then, too, the effects of alterations in data obtained through subjective evaluation (e.g., priority estimates) on the solution can be of interest.

Specifically, we propose to demonstrate a methodology that will provide insights into the Basic Staffing Model for the following four areas of concern:

1. The effect of altering the total number of hours of patient-centered activities provided (the "service level") on the objective function, staff mix, and required budget;

2. The impact of changes in the budget level on the objective function, staff mix, and service level;
3. The effect on the objective function, staff mix, and required budget of alterations in the priority-related demand estimates;
4. The impact on the objective function, staff mix, and required budget of changes in regulations concerning minimum staffing by skill level and/or staff-to-patient ratios.

Note that we may examine three subsidiary problems by means of the methods developed in connection with the first area. First, problems which are suboptimal with respect to the Basic Staffing Model optimal solution (for a given set of parameters) may be generated by restricting service levels to be less than that attained by the BSM optimum. The resulting staff mix and budget required to support the mix for each such problem can assist the administrator in recognizing a suboptimal situation in the existing facility staffing pattern. In addition, similar information may be helpful in planning a budget cut, since it would be easy to see the effect on total service and the objective function resulting from a decrease in the required budget. In this instance we simply interpret the result of the first subsidiary

problem in a different fashion. Finally, by relaxing the budget constraint and solving with alternative service levels, the effect of more funds on staff mix and the objective function may be examined. This allows for the consideration of the cost of providing increased care. A study of the results of the second area of concern allows the administrator to directly assess the impact of budgetary alterations. Both the service level and budgetary approach can provide valuable information for the specification of alternative courses of action, depending upon the orientation desired.

In order to obtain information with respect to the service level, the Basic Staffing Model is modified so that we may explicitly restrict the service level to be less than or equal to a constant, S. The resulting model, called Model I, is given as follows:

$$\text{MAX} \sum_{i}\sum_{j}\sum_{k} c_{ijk}x_{ijk} + \sum_{j}\sum_{k} p_{jk}\beta_{jk}$$

$$\text{s.t.} \sum_{i} s_i n_i \leq \text{Budget}$$

$$\sum_{j}\sum_{k} x_{ijk} - a_i n_i \leq 0, \qquad \forall i$$

$$\sum_{i} x_{ijk} - \beta_{jk} = 0, \qquad \forall j,k$$

$$\sum_{j}\sum_{k} \beta_{jk} \leq S \qquad \text{(III.9)}$$

$$L_i \leq n_i \leq U_i \text{ and integer}, \qquad \forall i$$

$$x_{ijk} \geq 0, \qquad \forall i,j,k$$

$$L_{jk} \leq \beta_{jk} \leq U_{jk}, \qquad \forall j,k$$

All quantities in Model I are as in the BSM. The constraint (III.9) is the necessary change. Obviously, we might solve Model I for each value of S, or budget level, of interest, such a tactic necessitating the solution of several mixed-integer linear programs. Obtaining solutions in this manner, however, can be a costly and time-consuming process. For this reason, we shall develop a methodology that produces solutions much more efficiently by exploiting the structure of the model. We defer consideration of this development to Chapter V.

We next turn to the consideration of obtaining information related to the final two areas of concern. Recall that the demand for nursing services for each care area and patient classification is essentially

determined by the model with β_{jk} a variable. Attached to each β_{jk}, however, are upper and lower bounds determined through priority related considerations. For a given mix of Int. B, Int. A and SNF patients determined by classification, the bounds on each of the three sets of care areas (indexed by k) are multiplied by the respective number of patients in each category. Thus, the resulting bounds yield, in the aggregate, the maximum and minimum demands for each j,k combination. Obviously, then, the third area of concern may be modeled by considering alterations in all or a subset of these bounding restrictions, given in the BSM as (III.8)

On the other hand, the final area of concern is usually related to the alterations in the minimum required number of personnel of various skill levels. Such changes may be incorporated by appropriate adjustments in the constraint set (III.6) of the BSM. Combining both of these concepts, which require the examination of alternative configurations of upper and lower bounds, we propose Model II as a reasonable characterization:

$$\text{MAX} \sum_i \sum_j \sum_k c_{ijk} x_{ijk} + \sum_j \sum_k p_{jk} \beta_{jk}$$

$$\text{s.t.} \sum_i s_i n_i \leq \text{Budget}$$

$$\sum_j \sum_k x_{ijk} - a_i n_i \leq 0, \qquad \forall i$$

$$\sum_i x_{ijk} - \beta_{jk} = 0, \qquad \forall j,k$$

$$\sum_j \sum_k \beta_{jk} \leq S \qquad \text{(III.10)}$$

$$L_i + \theta(L_i^* - L_i) \leq n_i \leq U_i + \alpha(U_i^* - U_i) \text{ and} \qquad \text{(III.11)}$$

$$\text{integer}, \qquad \forall i$$

$$x_{ijk} \geq 0, \qquad \forall i,j,k$$

$$L_{jk} + \theta(L_{jk}^* - L_{jk}) \leq \beta_{jk} \leq U_{jk} + (U_{jk}^* - U_{jk}) \; \forall j,k \qquad \text{(III.12)}$$

$$\theta, \alpha \in [0,1]$$

In constraint sets (III.11) and (III.12) the upper and lower bound quantities L_i^*, U_i^*, L_{jk}^*, and U_{jk}^* are chosen so that in varying θ and/or α on the interval [0,1] the desired range of consideration for the respective bounds is achieved. Constraint (III.9) of Model I has again been included as (III.10) in this most general form of Model II. In this instance, we may fix S *a priori* and examine alterations in bounds at any given service level.

In an analogous manner to that for Model I, the parameters θ and α could be fixed at values of interest,

and the resulting mixed-integer linear programs solved. Again, however, such a method would be tedious, necessitating a search for a more practical means of solution. The derivation of an algorithm for efficient solution of problems in the form of Model II will also be undertaken in Chapter V.

In summary, the extensions to the BSM incorporated into Model I and Model II capture the essence of the administrator's problem. Assuming that both models may be efficiciently solved, we have indicated how various interpretations of the results can produce valuable information both for the administrator and the director of nursing. In fact, the BSM need never be solved explicitly, its results being easily obtained, for example, as the initial problem in either Model I or Model II. An example of how the entire unified approach to staffing and the aggregate budget process, beginning with patient classification and ending with examination of the results of the two models, will be given in Chapter VI.

III.6 Data Requirements and Availability

In order to properly implement Models I and II, data for the following parameters are required:

1. Appropriateness measures, c_{ijk};
2. Availability constants, a_i;

3. Priority-related upper and lower bounds on demand for nursing service, L_{jk} and U_{jk};
4. Priority indicators, p_{jk};
5. Upper and lower bounds on the staff mix by skill level, L_i and U_i;
6. Salary costs by skill level, s_i;
7. Personnel budget allowance.

Since items 5, 6, and 7 are likely to be highly dependent on local conditions and regulations we will not discuss them further here. We shall, however, indicate sample values for these parameters in connection with an example of the methodology to be presented in Chapter VI.

The appropriateness measures, c_{ijk}, must be determined for every combination of skill level i performing the activities of care area j for patients of classification k. Obviously, such information is likely to be highly subjective, depending primarily on any given individual's perception of the appropriateness criterion. Having chosen to examine the issue of quality of care within our models, the appropriateness measures should reflect the relative efficiencies of assigning nurses with various degrees of professional expertise to the care of patients whose needs may differ considerably. Patient needs are obviously a function of both the overall degree of dependence (as captured in the

classification) and the specific care area in question. The concept of effective assignment subsumes the basic assumption that assignment of underqualified and over-qualified personnel with respect to a particular care area/classification combination is to be avoided.

Strictly speaking, the quantification of subjective perceptions of appropriateness should be executed so that interval-scaled c_{ijk} are obtained for use in our models. One such technique applicable in this instance is the Q-sort method demonstrated by Whiting [79]. This system essentially requires an evaluator to rank order the combinations of personnel and procedure from most appropriate to least appropriate by placing specified numbers of assignments into several discrete categories. The choice of intervals and required numbers per interval can lead to a discretized normal distribution, from which the interval ranking data are obtained. Liebman [38,39] applied this technique in deriving appropriateness measures for her staffing models, additionally comparing several transformations of the interval data for their relative properties. It was found that the set of ordinally scaled appropriateness coefficients produced results which were more in keeping with the evaluator's original perceptions of effectiveness than results obtained using other transformations.

Consequently, Liebman recommended the use of ordinally scaled preference measures in future staffing studies.

In order to derive such ordinal data for our models, the skill level preferences as reported by McKnight and Steorts [43] were utilized. Developed by a panel of three professional nurses, preferences are reported by patient nursing need as reflected in the tri-level classification (minimum, moderate, maximum care) for 71 specific nursing tasks. First and second choices of skill level usually are given. For purposes of completeness, the skill level category definitions which guided the panel are given below:

RN: Registered Professional Nurse - one who has successfully completed a program in an approved school of nursing and who is licensed to practice nursing as a registered nurse in the state in which she is employed.

LPN: Licensed Practical Nurse - (a) one who is a graduate of an approved school of practical nursing and is duly licensed under the provisions of the Practical Nurse Act of the state in which she is employed; (b) one who has received a license by waiver or examination as provided for in the Practical Nurse Act of the state in which she is employed.

NA: Nursing Assistant - one who is employed as an "auxiliary personnel" in the nursing service of a health facility to assist the nurse. These persons are employed and trained to perform tasks which involve specified services for patients as delegated by the professional nurse and performed under the supervision of a professional nurse or licensed practical nurse.

In order that compatibility between these results and results by care areas might be obtained, the tasks subsumed by each care area were identified, and the preferences for each task examined collectively. Although in most cases the overall care area preference structure was apparent, professional judgment* was employed to indicate the structure for those cases that had no readily apparent solution. Scoring a first preference as 2 and a second preference as 1, the resultant appropriateness measures are shown in Table III.7.

The second set of parameters of the staffing models which must be obtained are the availability constants, a_i. The manner of presentation of these quantities is certainly dependent on the time frame and employment status (full or part time) chosen for consideration. Recall, however, that we shall assume a 24-hour time

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* Consultation with Eleanor McKnight, R.N., M.P.H.

Table III.7

Ordinal Skill Level Preferences by Care Area and Nursing Need, Based on the Study of McKnight and Steorts [43]

Care Area	Minimum	Moderate	Maximum
1. Elimination	NA-2	NA-2	LPN-2
	LPN-1	LPN-1	RN-1
2. Catheterization	RN-2	RN-2	RN-2
3. Daily Care	NA-2	NA-2	RN-2
	LPN-1	LPN-1	LPN-1
4. Bathing	NA-2	NA-2	RN-2
	LPN-1	LPN-1	LPN-1
5. Bedmaking	NA-2	NA-2	RN-2
	LPN-1	LPN-1	LPN-1
6. Care of Skin	NA-2	LPN-2	RN-2
	LPN-1	RN-1	LPN-1
7. Care of Hair and Face	NA-2	NA-2	NA-2
	LPN-1	LPN-1	LPN-1
8. Oral Hygiene	NA-2	NA-2	RN-2
	LPN-1	LPN-1	LPN-1
9. Care of Nails	LPN-2	LPN-2	LPN-2
	NA-1	NA-1	NA-1
10. Rehabilitation and	NA-2	NA-2	RN-2
Recreation	LPN-1	LPN-1	LPN-1
11. Food and Nourishment	NA-2	NA-2	NA-2
12. Medications	RN-2	RN-2	RN-2
	LPN-1	LPN-1	LPN-1
13. Cardinal Symptoms	LPN-2	LPN-2	LPN-2
	RN-1	RN-1	RN-1
14. Therapeutic Measures	RN-2	RN-2	RN-2
	LPN-1	LPN-1	LPN-1
15. Urine Tests	LPN-2	LPN-2	LPN-2
	RN-1	RN-1	RN-1
16. Enema	NA-2	NA-2	LPN-2
	LPN-1	LPN-1	RN-1
17. Care of Colostomy or	LPN-2	LPN-2	LPN-2
Ileostomy	RN-1	RN-1	RN-1
18. Associated Nursing	RN-2	RN-2	RN-2
Care Functions	LPN-1	LPN-1	LPN-1
19. Weighing Patient	NA-2	NA-2	RN-2
	LPN-1	LPN-1	LPN-1
20. Gavaging Patient	RN-2	RN-2	RN-2
21. Female Procedures	NA-2	NA-2	LPN-2
	LPN-1	LPN-1	NA-1
22. Oxygen Therapy	RN-2	RN-2	RN-2
23. Care of Critically Ill	RN-2	RN-2	RN-2
	LPN-1	LPN-1	LPN-1
24. Death of Patient	NA-2	NA-2	NA-2
25. Miscellaneous	NA-2	NA-2	NA-2

span for the model with no part-time personnel allowed. The models then determine the number of nurses, by skill level, working an 8-hour shift, with shift specification for individuals being left to the discretion of the director of nursing. We would like to know, therefore, the portion of a shift, averaged over all three shifts, that personnel of each skill level usually have available to devote to patient-centered activities.

An indication of these availabilities was obtained by examination of the results of a nursing work sampling study concurrently undertaken by McKnight [42] in connection with her Colorado work. Data were obtained in four homes of various sizes related to the partitioning of nursing time into five mutually exclusive activity areas:

1. Patient centered;
2. Personnel centered (training, staff development);
3. Unit centered (environment, supplies, etc.);
4. Home centered;
5. Other centered (personal, standby, relief).

Patient-centered activities being more or less analogous to those tasks subsumed by the 25 care areas, the

percentage of time, averaged over all shifts, for these activities was obtained. When considering these data categorized by nursing skill level, both charge and staff RN's and LPN's were included. The resulting percentages and availabilities per eight-hour shift are shown in Table III.8. Two factors were noted in the

Table III.8

Percentages of Time (Averaged Over Three Shifts) and Number of Hours Available Per 8-Hour Shift for Patient-Centered Activities

Skill Level	Percentage	Availability/8-hours
RN	69.9	5.6
LPN	72.1	5.7
NA	56.6	4.5

derivation of these results. First, home size (as indicated by the number of units) had an obvious effect on the distribution of professional nursing time. That is, in the smaller facilities professional nurses typically gave more time to patient-centered activities and less to supervisory work than in the larger facilities. Second, a significant portion of the non-professional nurses' time was being given to unit and other centered actitivies.

We next consider the upper and lower bounds on the demand variables β_{jk}. Recall that in addition to allowing flexibility in meetings patient demands, the limits are set with consideration being given to the priority of certain care areas over others within and between patient classification levels. Using the average daily required performance time data shown in Table III.6, professional assistance* was again enlisted to derive a sample set of limits. Adjustments in these bounds may be made by utilizing the properties of Model II, but the bounds proposed at least serve as a basis for examination of adjusted estimates.

One factor in attempting to consider priorities as related to the bounds was the number of patients, by classification, who actually received care in connection with the activities of each care area. Assuming the distribution of activities among this patient sample is fairly representative of the distribution among patients of similar nursing needs, this input more or less reflects the priorities existing on the units at the time of the original study. Taken together with more normative concepts of priorities of care, these considerations formed part of the basis for setting bounds.

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* Consultation with Eleanor McKnight, R.N., M.P.H.

Table III.9 below indicates the percentage of patients of each category receiving care and the upper and lower bounds for each care area. Note that information for care areas 20 and 24 is omitted due to lack of observed activities. For other care area/classification combinations not observed in the original study but for which partial information did exist, an estimate was made.

By abstracting the information contained in Table III.9 the average and extreme daily care times by patient classification may be derived. These results are presented in Table III.10. As can be seen, the low and average times are nearly equal, with a definite tendency towards allowing for greater staffing requirements.

Finally, we present an indication of the priority structure for care area/patient classification combinations in the last column of Table III.9. Recall that $p_{jk} = 1$ indicates a relatively high priority, and $p_{jk} = 0$ a lower relative priority.

Table III.9

Upper and Lower Bound on β_{jk} (in minutes), Percentage of Patients in Each Category Receiving Care, and Priority Constants p_{jk} (N_{min}=64, N_{mod}=95, N_{max}=36)

Care Area	Class.	%Patients	Average	Lower	Upper	p_{jk}
1. Elimination	min	14	.53	.33	.66	0
	mod	60	4.94	3.75	5.50	0
	max	72	8.24	9.00	9.50	1
2. Catheterization	min	2	.02	.01	.03	0
	mod	6	1.0	.75	1.50	0
	max	33	5.84	5.50	7.00	1
3. Daily Care	min	89	2.36	2.25	3.50	1
	mod	72	23.23	23.00	26.00	1
	max	89	43.79	40.00	45.00	0
4. Bathing	min	11	2.59	2.25	3.25	1
	mod	21	8.09	7.50	9.00	1
	max	31	13.94	13.50	17.00	1
5. Bedmaking	min	55	3.46	3.00	4.00	0
	mod	81	6.11	5.75	7.50	0
	max	69	6.41	6.00	8.50	1
6. Care of Skin	min	2	.05	.03	.06	0
	mod	25	1.79	1.50	2.75	0
	max	78	11.10	11.00	15.00	1
7. Care of Hair and Face	min	28	.75	.60	.85	0
	mod	79	2.89	2.25	4.00	0
	max	94	3.02	3.00	5.00	1
8. Oral Hygiene	min	9	.02	.01	.03	0
	mod	29	.58	.50	1.00	0
	max	56	1.20	1.00	2.50	0
9. Care of Nails	min	9	.13	.10	.15	0
	mod	17	.35	.30	.45	0
	max	19	.35	.30	.75	0
10. Rehabilitation and Recreation	min	53	1.31	1.25	2.50	1
	mod	69	4.61	4.50	7.50	1
	max	69	6.68	6.50	10.00	1
11. Food and Nourishment	min	98	8.56	7.50	9.50	0
	mod	100	17.35	16.50	20.00	0
	max	100	36.32	35.00	45.00	0
12. Medications	min	91	5.00	5.00	7.50	1
	mod	88	9.27	8.50	10.00	0
	max	82	10.45	10.00	14.00	1
13. Cardinal Symptoms	min	8	.61	.50	.75	0
	mod	17	.90	.75	1.75	0
	max	28	2.90	2.50	4.50	0
14. Therapeutic Measures	min	3	.04	.03	.06	0
	mod	8	.44	.40	.75	0
	max	11	1.92	1.50	3.00	0
15. Urine Tests	min	2	.17	.15	.30	0
	mod	8	.49	.40	.75	0
	max	8	.18	.15	.50	0

Table III.9, Cont.

Care Area	Class.	% Patients	Average	Lower	Upper	p_{jk}
16. Enema	min	2	.12	.10	.25	0
	mod	8	1.00	.75	1.50	0
	max	17	1.00	.75	2.00	0
17. Care of	min	0	0	0	0	0
Colostomy or	mod	1	.21	1.25	4.25	0
Ileostomy	max	0	0	1.25	4.25	0
18. Associated	min	100	4.13	3.25	5.25	0
Nursing Care	mod	100	6.17	5.75	7.25	0
Functions	max	100	6.21	6.00	8.50	0
19. Weighing Patient	min	9	.04	.03	.05	0
	mod	5	.04	.03	.05	0
	max	0	0	.05	.10	0
20. Gavaging Patient - Omitted						
21. Female Procedures	min	0	0	.01	.04	0
	mod	1	.02	.01	.04	0
	max	0	0	.01	.04	0
22. Oxygen Therapy	min	2	.12	.05	.15	0
	mod	2	.09	.07	.10	0
	max	6	.16	.15	.30	1
23. Care of	min	2	.20	.10	.25	0
Critically Ill	mod	1	.11	.10	.30	1
	max	0	0	.15	.50	1
24. Death of Patient - Omitted						
25. Miscellaneous	min	100	5.05	4.50	5.75	0
	mod	100	5.50	5.00	6.50	1
	max	100	6.50	6.50	7.50	1

Table III.10

Average and Extreme Daily (24-hour) Patient Staffing Requirements (in hours) by Patient Classification

Patient Classification	Lower Bound	Average	Upper Bound
Minimum	.52	.61	.75
Moderate	1.49	1.55	1.99
Maximum	2.65	2.77	3.51

Chapter IV. Patient Classification for Long-Term Care

IV.1 Introduction

"...Patterns are the means by which we interpret the world." (Meisel [45, p. 1])

Having explored in general terms in the foregoing chapter the question of patient classification in the long-term care setting, we now move to a more detailed treatment of the topic. First and foremost, we seek to demonstrate a methodology that will be attractive computationally both for analytical purposes and for future implementation. The application of such a method to the best data currently available and the judicious use of the results obtained will lead us to a system which meets the criteria for an acceptable patient classification system established earlier. Recall that the essence of these guidelines was to categorize patients based on an assessment of their individual medical, psycho-social, and functioning status, and then to relate the classification to demand for nursing services. As an addendum to the above, the classification system sought should be applicable in as efficient and least time-consuming a manner as possible. Having previously demonstrated how any tri-level system may be related to demand, we

concentrate here on derivation of a specific classification methodology.

The results of previous studies discussed in Chapter II are, for the most part, lacking with respect to our criteria. Most are obviously unsuitable for relating the aforementioned factors to a prescribed level of care and thence to demand for service. In those studies where the relationships could be inferred, either new data have subsequently been obtained or the operational characteristics of the method with respect to facility level use were not explored in depth. None of this is said by way of criticism of work that has preceded this study; rather, it tends to demonstrate that no classification system will be universally acclaimed for all possible uses, and so we proceed.

Initially, some general ideas on the generic class of problems known as pattern recognition will be discussed, followed by a brief overview of specific techniques applicable in solving such problems. After demonstrating the derivation of an existing technique due to Walker and Duncan [78] and a discussion of its pertinence to the current study, application of the method to the data will be undertaken, and a readily applied procedure will be proposed. Comparisons with past studies and some recommendations for future research will conclude

the chapter.

IV.2 The Pattern Recognition Problem

Researchers in the behavioral, social, and physical sciences are often confronted with large amounts of data which are typically multivariate in nature. Fisher, Kronmal, and Diehr [23] point out that the medical sciences also tend to generate information of this type, characteristically even more multivariate due to the nature of the field. All of these groups share the common goals of gaining an understanding of the underlying mechanisms of the animate or inanimate system which produced the data and becoming adept at analyzing new data in terms of historical precedents. Both goals are the province of the so-called pattern recognition problem.

In an excellent treatment of the subject of pattern recognition, Meisel [45] points out that the major aspects of any such study are the detection of possible regularities in the data and the synthesis of such information as may be available into a decision rule to be used in analyzing future data. This is the "recognition" aspect of the problem. Note, however, that the application of the rule need not only occur *ex post facto*, but may be made an integral part of the

derivation of the rule itself. This latter contingency is called "learning and recognition concurrently," the former being simply, "learning before recognition." Both cases are shown schematically in Figures IV.1 and IV.2 which have been reproduced from Meisel [45, p. 6].

Figure IV.1

Learning Before Recognition

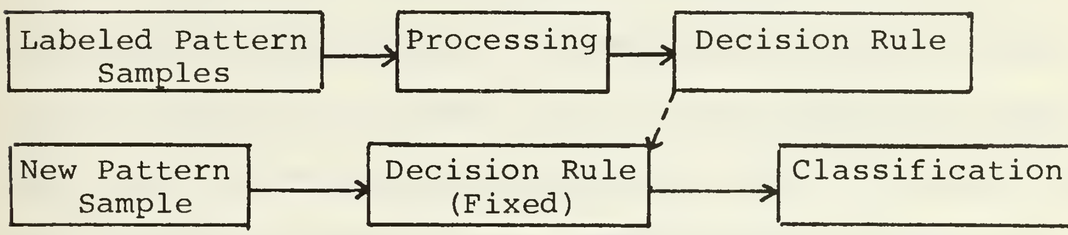
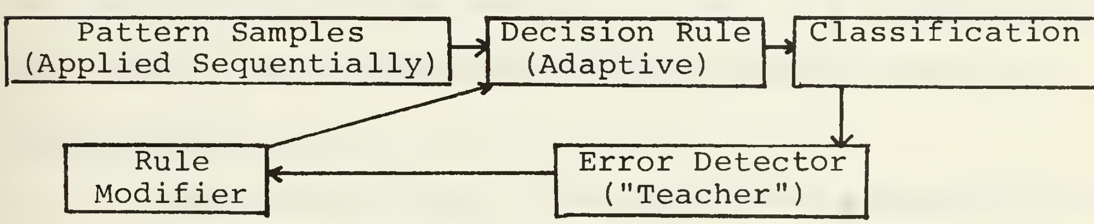


Figure IV.2

Learning and Recognition Concurrently



The concepts of learning and recognition are identified by Sebestyen [64, p. 5] as being the key points of pattern recognition. The former is said to be, "...the estimation of the probability densities describing the distribution of the set of samples in the vector space," while the latter is, "...based on the evaluation of the

already learned conditional probability density of each class at the point in the vector space that represents the new input to be classified."

Meisel goes on to point out, however, that typically the decision rule obtained via either of the above schemes is of academic interest only, primarily because the dimensionality of the problem (corresponding to the number of variables under consideration) is too large for practical usage. Certainly the advent of the digital computer has made work in such dimensions tractable, but reduction of the data to a lower dimensionality is a pleasing concept from a computational as well as implementational point of view. This is especially true when the cost of obtaining information for further study or installation of the results is high. Some definitions will assist us in understanding the concepts presented.

Definitions [45]

Measurement Space: The space with dimensionality corresponding to the number of variables under consideration initially.

Pattern Space: A finite-dimensional space of relatively low dimension compared to the measurement space. The corresponding variables contain sufficient information to perform the classification process.

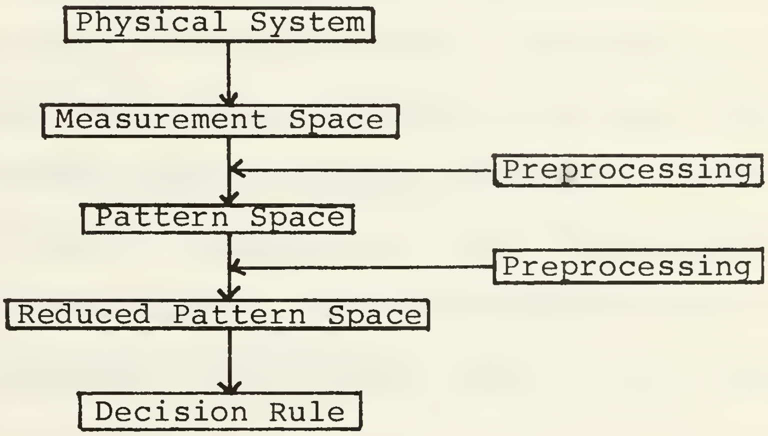
Feature Selection: (Preprocessing): The determination of the components of the pattern space.

Pattern Classification: The derivation of a decision rule to classify an unlabeled sample in the pattern space.

Given the modified framework above, the pattern recognition problem may now be viewed as a combination of feature selection and pattern classification. Figure IV.3 taken from Meisel [45, p. 7] shows in flow chart form the revised pattern recognition process.

Figure IV.3

Stages in the Derivation of the Decision Rule



The process of feature selection is therefore central to the concept of pattern recognition. In terms

of the current study it is obvious that feature selection will be an appropriate approach to solving the problems posed by the second of our criteria. Meisel points out several general attributes which an acceptable set of features must possess. These include the following:

1. Low dimensionality;
2. Sufficient information to yield recognition of patterns;
3. Geometric consistency (preservation of order and distance concepts);
4. Consistency of features throughout the samples.

Even having determined a set or sets of features satisfying Meisel's criteria we have yet to solve the problem, for there almost certainly exists a dominance of some feature sets over others in terms of how well they perform the ultimate pattern recognition task. The determination of the best feature set falls within the purview of the "optimal subset (feature) selection" problem and will be discussed in more detail later in this chapter.

A final concept which Meisel discusses is the quantification of the decision rule obtained from our process. In general, two methods are available. The

first is the specification of a boundary function such as

$$b(\underline{x}) = 0 \qquad \text{(IV.1)}$$

where $\underline{x}$ is taken to be the vector of features for an unlabeled point. The decision rule simply stated is to assign the point to group one if (IV.1) is non-negative or to group two if (IV.1) is non-positive. Points falling on the boundary may be assigned arbitrarily. For N groups, the derivation of

$$\frac{N(N-1)}{2} \qquad \text{(IV.2)}$$

such boundaries is shown to be sufficient. The second approach uses vector functions of the features, one for each of the groups, called $p_1(\underline{x})$, $p_2(\underline{x}), \ldots, p_n(\underline{x})$. The rule is to classify the new point as a member of group i if

$$p_i(\underline{x}) > p_j(\underline{x}), \qquad \forall\, j \neq i \qquad \text{(IV.3)}$$

This method is most often associated with the technique of discriminant analysis which will be discussed below. It will also be shown to be a more general concept applicable to a number of multivariate techniques.

Before concluding the discussion of pattern recognition, a few additional words on the reduction of dimensionality problems are in order. Most of the multivariate statistical techniques in general use today,

such as principal components analysis, factor analysis, discriminant analysis, cluster analysis, and multiple linear regression embody the concept of feature selection as either their primary aim or a secondary method. More detail on these methods will be provided in the following section, but for now it is sufficient to say that there exists a number of mechanical procedures to deal with the problem in an efficient computational manner. Fisher, Kronmal, and Diehr [23], however, emphasize the point that, "Since the goal is to classify, and not to demonstrate methodology, one need not restrict the reduction of data to objective methods" [23, p. 381]. They further state that, "Rather than depend upon the sheer computational power of a computer, all available scientific knowledge should be used to reduce the task" [23, p. 382]. In short, the authors argue that beyond the techniques mentioned above, the application of intuition, expert opinion, and experience gained in previous studies is often the best way to extract features. One caveat that must be attached to their argument, however, is to be wary of confusing the predictive powers of a feature set with a causal relationship to the classification. Care must be taken that this point is understood by those from whom intuitive information is solicited. In any event, the use of informed

opinion is invaluable not only in direct feature extraction, but also in interpretation of results obtained by mechanical processes.

IV.3 A Review of Multivariate Statistical Techniques

We have seen that the patient classification problem posed earlier is of the pattern recognition form, specifically requiring the steps of feature extraction and pattern classification. The several statistical techniques that lend themselves either directly or indirectly to the solution of the problem each have certain attributes that make them possible candidates for our use. However, the necessary assumptions and restrictions each possesses should make us wary of using them indiscriminantely, and for this reason we will examine each for its strengths and weaknesses vis-a-vis the current problem.

The method of principal components analysis is an often-used approach to feature extraction. The essence of the technique is the extraction from the set of original variables a subset of lower dimension that nevertheless explains a significant proportion of the total variation in the data sample. Beginning with no distributional assumption on the vector of variables, $\underline{x}$, Lawley and Maxwell [37] show that if we take the matrix

A as the variance-covariance matrix of the sample with eigenvalues

$$\lambda_1, \lambda_2, \ldots, \lambda_p \tag{IV.4}$$

where A is (p x p), and if we also have the associated eigenvectors of A as

$$\underline{\mu}_1, \underline{\mu}_2, \ldots, \underline{\mu}_p \tag{IV.5}$$

such that

$$\underline{\mu}_i' \underline{\mu}_j = 0 \qquad \forall\ i \neq j \tag{IV.6}$$

$$\underline{\mu}_i' \underline{\mu}_i = 1$$

then the principal components of a new vector y can be written as

$$\underline{\mu}_1'\underline{y}, \ldots, \underline{\mu}_p'\underline{y}. \tag{IV.7}$$

Assuming that

$$E(\underline{y}) = \underline{\mu} \tag{IV.8}$$

$$\mathrm{Var}(\underline{y}) = A$$

then we may write

$$\underline{y} = \sum_{i=1}^{p} \underline{\mu}_i \underline{\mu}_i' \underline{y} = \sum_{i=1}^{p} z_i \underline{\mu}_i \tag{IV.9}$$

where the z_i are uncorrelated, implying that the y_i can be represented as a weighted combination of orthogonal vectors $\underline{\mu}_i$. Noting that we can write

$$U'AU = \Lambda \tag{IV.10}$$

where U is the matrix whose columns are the eigenvectors $\underline{\mu}_i$ and Λ is a diagonal matrix with λ_i, i=1,...,p on the

diagonal, and further assuming that the λ_i have been ordered by decreasing magnitude, the following result is obtained. With

$$tr(A) = \sum_{i=1}^{p} \lambda_i \tag{IV.11}$$

being a good characterization of the variance of $\underline{y}$, then

$$\sum_{i=1}^{P} \lambda_i - \sum_{i=1}^{k} \lambda_i, \qquad k < p \tag{IV.12}$$

small implies that the k corresponding uncorrelated variables

$$z_i = \underline{\mu}_i' \underline{y}, \qquad i=1,2,\ldots,k \tag{IV.13}$$

explain most of the variation in the sample, and hence are a good set of features to use. If we are willing to make the distributional assumption of multivariate normality on $\underline{x}$ then tests of the hypothesis that the remaining p-k eigenvalues are essentially equal but non-zero may be made to enable the conclusion that the residual variation is split equally among the remaining variables. Although principal components analysis provides a set of features it does not allow for the direct completion of the classification process, since no dependent variables are involved. It would, therefore, probably have to be coupled with another method (e.g., discriminant analysis) which would classify based on principal components only. Additionally, the method is

not invariant with respect to difference in scales for the variables of the sample, a drawback that presents difficulties in the case of the current study.

A method that does not suffer from this last distraction is factor analysis, an extension of the concepts of principal components. According to Harman [29], the classical factor analytic model may be written as

$$z_j = a_{j1}F_1 + a_{j2}F_2 + \dots + a_{jm}F_m + d_jU_j, \quad j=1,\dots,n \qquad \text{(IV.14)}$$

where n is the number of observed variables which are linearly described in terms of m (<n) common factors F_i and a unique factor U_j. The a_{ji} are called factor loadings. The method thus represents the observed variables in terms of a weighted linear combination of common latent variables (factors) of lesser dimensionality than the original space, plus a specific factor accounting for variability which cannot be attributed to the common factors. In matrix terms we may write (IV.14) as

$$\underline{z} = \Lambda \underline{F} + \underline{\epsilon} \qquad \text{(IV.15)}$$

where Λ is the matrix of factor loadings, $\underline{F}$ is the vector of common factors, and $\underline{\epsilon}$ an error vector which here includes both the specific variability and any unreliability. Now obviously one must specify in advance the number of factors, k, believed to give the most reasonable form to (IV.15), one possible approach being to

increase k until no appreciable alteration in the amount of variation explained is forthcoming. With this in mind, we can determine Λ in (IV.15) from a form of

$$\Sigma = \Lambda\Lambda' + \Psi \tag{IV.16}$$

where, according to Morrison [46], Σ is the variance-covariance matrix of the sample and Ψ is the diagonal matrix with the errors ϵ_i on the diagonal. Lawley and Maxwell [37] recommend the use of the method of maximum likelihood to determine Λ and Ψ. They go on to point out, however, that for $k>1$ we may not specify the matrix Λ uniquely, and therein lies both the flexibility and difficulty with this technique.

We would like the latent variables to allow an intuitively appealing interpretation in terms of the original variables, with the loadings representing the amount of variation of an original variable explained by the corresponding latent variable. To this end, and taking advantage of the non-uniqueness property of Λ, a series of transformations on Λ may be undertaken to yield a reasonable form. These transformations are called rotations and may be described mathematically by

$$\Lambda_{i+1} = \Lambda_i R_{i+1} \tag{IV.17}$$

where R_{i+1} is the appropriate rotation matrix. Naturally there is a certain amount of subjectivity in choosing R_{i+1}, causing Harman [29] to speak of the art

involved in the method at this point. Some commonly accepted structures do exist, however, that seem reasonable from an interpretive standpoint. One of these is rotation to "positive manifold," which simply stated is the application of the linear transformation to make all elements of Λ non-negative. The rationale is that if we choose to use the correlation matrix in place of Σ in (IV.16), then we would not expect the loadings generally to be negative in any reasonable Λ. A second possible transformation is rotation to "simple structure," wherein there are only a few non-zero elements in any row and column of Λ, and the pattern of loadings is significantly different for each factor. Comrey [17] points out that such a structure allows for more intuitive interpretation of the latent variables in terms of the original sample variables. A final type of transformation is a procrustean rotation, wherein an attempt is made to force the loading matrix to fit some preconceived target form developed from a subjective estimation of the pattern of loadings. Numerous computational schemes have been devised to perform these transformations, the major differences in them being the ultimate structure sought and the orthogonality or obliqueness of the factors themselves.

Certainly factor analysis is a reasonable

technique for feature extraction, the features being the latent variables. The method is highly subjective, however, and like principal components analysis does not lead directly to an operational classification without an intermediate application of another method. In general, no distributional assumptions are made with regard to the sample, but applications of some of the advanced techniques suggested by Lawley and Maxwell [37] for testing hypotheses concerning the number of factors do require such an assumption.

A third possible technique for use in the pattern recognition problem is cluster analysis, a method very popular in biological taxonomy for a number of years. Meisel [45] differentiates between clustering methods and pattern classification on the basis of the fact that the samples used in the former are unlabeled, the goal being to determine natural groupings within the sample that might correspond to natural groupings in the population. Tryon and Bailey [73] call cluster analysis a "poor man's factor analysis," its methods being based more upon simple logic than esoteric mathematics. The clustering of variables into groups has an analogous interpretation to that of factor analysis, in that the clusters roughly correspond to the latent variables and as such require interpretation. Anderberg [2] suggests

that after deciding on the data to be used and variables to be included, a choice of clustering data units or variables must be made, followed by the choice of similarity measures, clustering criteria, and the number of clusters desired. As in principal components analysis some care must be taken with the scaling of variables to avoid nonsensical results. A usual technique is standardization to zero mean and unit variance of all variables. At this point there is a wide divergence in the specific algorithms available, depending largely upon whether one wishes to cluster variables or data units. Initially, we determine a convenient similarity measure to use. In the case of clustering of variables, the correlation matrix is computationally attractive. It should be noted that the specific type of correlation used will be a function of the scaling of the variables as ratio, interval, ordinal, or nominal. Anderberg suggests methods by which diverse mixtures of scales can be made compatible. For data units, convenient similarity measures (which are the elements of a similarity matrix S) are Euclidean distance

$$D(\underline{x},\underline{y}) = [\sum_{i=1}^{n} (x_{ij}-y_{ij})^2]^{1/2} \qquad \text{(IV.18)}$$

where n is the number of data units, or the "city block" distance function

$$D(\underline{x},\underline{y}) = \sum_{i=1}^{n} \left| x_{ij} - y_{ij} \right| \qquad \text{(IV.19)}$$

both of which satisfy the necessary condition that the measure be a metric. Taking our lead from Anderberg, the major classes of algorithms are the hierarchical and nonhierarchical methods, the latter being useful only in clustering data units. Among the hierarchical methods, agglomerative and devisive techniques are the most prominent. Agglomerative algorithms move from clusters of one unit each to a final stage where one cluster of all n units is formed. Intermediate stages are identified by various numbers of clusters. The devisive algorithms perform in the opposite fashion. For the sake of illustration, a basic agglomerative algorithm due to Anderberg [2, p. 133] is presented:

Step 1: Begin with n clusters each consisting of exactly one entity. Let the clusters be labeled 1 through n.

Step 2: Search the similarity matrix S for the most similar pair of clusters. Let the chosen clusters be labeled p and q and let their associated similarity be s_{pq}, $p>q$.

Step 3. Reduce the number of clusters by 1 through merger of clusters p and q. Label the product of the merger q and update the

similarity matrix entries in order to reflect the revised similarities between cluster q and all other existing clusters. Delete the row and column of S pertaining to cluster p.

Step 4. Perform steps 2 and 3 a total of n-1 times (at which point all entitites will be in one cluster). At each stage record the identity of the clusters which are merged and the value of the similarity between them in order to have a complete record of the results.

A widely used variant of this algorithm chooses Euclidean distance as the similarity measure and the centroids of the new clusters formed as the points of reference. Devisive algorithms cause successive splits in the set of data units until n clusters of cardinality one remain. Little will be said about nonhierarchical methods, save for the fact that, in general, they allow for the specification in advance of the ultimate number of clusters desired. For additional insights into comparative measures of performance and application of the various algorithms, the reader is referred to Gower [27] and Slagle, Chang, and Lee [69].

In retrospect, a cluster analysis on the variables of the sample would lead to feature extraction,

while analysis on the data units would generate natural classes. Attempts to cluster data units corresponding to a sample of patient assessment records from the CPAI were made in connection with the current study. Although appealing, this approach was found to be unacceptable both from the standpoint of compatability of results and the cluster membership derived from the data analysis. Additionally, the problem of functionally relating the features to the classes remains as in the previous methodologies explored. Specifically, clear discrimination between the clusters obtained in terms of level of care of member units could not be obtained. This effectively rendered the results unusable for determining classification rules. As was noted in Chapter II, Parker and Boyd [53] were able to achieve acceptable results using a superset of the data currently under study by adding the centroid scores of those variables found to be most important from the "B/T criterion" and relating the sums to cluster membership. A retrospective study on the data using this rule is not reported, however. Attempts in connection with the current study to apply such a rule were found to yield erratic and unacceptable results.

A fourth branch of statistical methodology for our consideration is discriminant analysis, by which one

may ascribe a prior dependent variable to correspond to the labeled classification of each data vector $\underline{x}$ and derive a linear function of $\underline{x}$,

$$y = \underline{wx} \quad \text{(IV.20)}$$

The vector $\underline{w}$ is a weighting vector and must be determined from the data. It can be shown in the dichotomous case that

$$\underline{w} = (S^2)^{-1}\underline{d} \quad \text{(IV.21)}$$

where $\underline{d} = \bar{\underline{x}}_1 - \bar{\underline{x}}_2$ and S is the sample variance-covariance matrix will yield a function (IV.20) such that the variance between the two groups in question is maximized. Such a criterion is reasonable from the standpoint of choosing $\underline{w}$ such that maximum discrimination may be made between groups one and two. The method has extensions to the k group case where k-1 functions of the form (IV.20) must be specified. The classification rule which may be derived from discriminant analysis in the general case is to assign the data unit, $\underline{x}$, to group i if the value of (IV.20) at $\underline{x}$ is closer to (IV.20) evaluated at the centroid of group i than the value of (IV.20) at the centroid of group j, $j \neq i$.

Discriminant analysis thus provides an acceptable classification rule which may be readily applied in accordance with our second criterion. The question of feature selection is not so straightforward. One

approach is to consider the magnitude of the elements of $\underline{w}$ as being indicative of the effect each variable has on the ultimate classification. This tactic may only be useful up to that point at which there exists an inordinate number of variables with similar w_i. How to choose among them is still an open question, with the optimal subset selection problem also coming into play. An additional note on the method is that any ordering information concerning the dependent variables is ignored, thus eliminating its potential use. The results of Parker and Boyd [53] reported in Chapter II are nevertheless an encouraging application of the method, albeit some 60 variables were used to effect the classification.

The widely used techniques of multiple linear regression (MLR) have met with great success, under the appropriate conditions, in handling the problems of classification and feature selection. The basic model is

$$\underline{y} = X\underline{\beta} + \underline{\epsilon} \tag{IV.22}$$

where $\underline{y}$ is the vector of dependent variables (n x 1), X the data matrix [nx(p+1)], $\underline{\beta}$ the vector of parameters to be estimated [(p+1) x1], and $\underline{\epsilon}$ the error vector (n x 1). The method of maximum likelihood can be applied to obtain estimates of $\underline{\beta}$, called $\underline{b}$ such that

$$\underline{b} = (X'X)^{-1}X'\underline{y} \ , \tag{IV.23}$$

the model (IV.22) thus being fitted and providing a good

classification tool. The theory of MLR has been sufficiently advanced to allow us to make some reasonable attempts at feature selection once (IV.23) has been obtained. Draper and Smith [22] point out four ways in which this may be done. In each case an attempt is made to find a smallest subset of variables without significantly degrading the predictive power of the model. The four methods are the following:

1. Examine all possible combination of variables; there exist 2^p such combinations in individual regression models.
2. Backward elimination - Solve the model in p variables. Test the hypothesis that each b_i obtained is significantly different from zero using an F- or t-test. Eliminate those found to be insignificant, resolve and repeat until all coefficients are found to be significant.
3. Forward selection - Test each variable before entering it into the model using a partial F-test to see if it accounts for a significant amount of variation using R^2 as the criterion. Stop when no such variables can be found.
4. Stepwise regression - Similar to forward selection, except at each step of the process

all variables in the model are reexamined to see if they are still significant using the partial F-test, as well as performing the function in (3). Variables which do not meet the level of significance are either dropped or not added as the case may be.

With its predictive capabilities and selection possibilities, MLR is the most appealing method examined to this point. The assumptions of multivariate normality of the data can often be justified using Central Limit Theorem arguments for large sample sizes. The disturbing fact, however, is that the dependent variable is assumed to be a continuous, linear function of $\underline{x}$, a supposition that is actually unwarranted in a situation where we are attempting to fit a truly polychotomous response. Although the ordering information on the dependent variable will now be important, something more than an ordinal relationship will be implicitly assumed. That is, typically we might assign the value 1 and 2 to the response variable for two groups, with the obvious ordinal relationship implied. But MLR takes such an assignment as an interval measure, when in actuality interval values (if they could be obtained) might be 1 and 10, for example. Additionally, the assumed

continuity of $\underline{y}$ can often lead to "fuzzy" classifications such as 1.4 between 1 and 2, in which case one is hard pressed to make the assignment rule mathematically tractable. We conclude, therefore, that MLR probably comes closest to a reasonable technique for our use, but its limitations for present purposes are serious enough to warrant further exploration.

As a final point, we reiterate the Bayesian approach of Parker [52] for dichotomous independent variables. Recall that, from Bayes' Theorem, the basic classification rule is determined from

$$P(d_i \mid \underline{S}) = \frac{P(\underline{S} \mid d_i)P(d_i)}{\sum_{i=1}^{m} P(\underline{S} \mid d_i)P(d_i)} \qquad \text{(IV.24)}$$

with d_i, $i=1,\ldots,m$, indicating the categories and $\underline{S}$ being the binary vector representing a patient profile on the n independent variables. Further,

$$P(\underline{S} \mid d_i) \doteq P(S_1 \mid d_i) \cdot P(S_2 \mid S_1 \cap d_i) \cdot \ldots \cdot P(S_n \mid S_{n-1} \cap d_i). \qquad \text{(IV.25)}$$

The rule is to assign the patient with profile $\underline{S}$ to category i if

$$P(d_i \mid \underline{S}) > P(d_j \mid \underline{S}), \qquad j \neq i . \qquad \text{(IV.26)}$$

Parker went on to perform a feature selection procedure using two *ad hoc* measures that indicated the ability of the original variables to separate the groups, and his results were quite reasonable. The Bayesian approach is

certainly useful in capturing the discrete nature of the ordered dependent variable, and methods such as those proposed by Parker to deal with feature selection seem apropos. That the Bayesian approach could be extended for use with polychotomous independent variables is obvious, but the ordinal information in the variables would not be fully exploited, and the computations would be tedious though easy. In summary, the method seems best applied to binary $\underline{x}$, a fact emphasized by Parker.

IV.4 The Method of Walker and Duncan

The foregoing section serves to point out the basic attributes and deficiencies of the methods surveyed. A major point that could be gleaned from the summaries presented is that if the strengths of discriminant analysis in the classification realm and the techniques of feature extraction developed in conjunction with multiple linear regression could be melded into a single method embodying each, a satisfactory answer to the pattern recognition problem currently under consideration might be achieved. It is submitted that such a method is one developed by Walker and Duncan [78].

The essential feature of this method is the modeling of the polychotomous response variable in terms

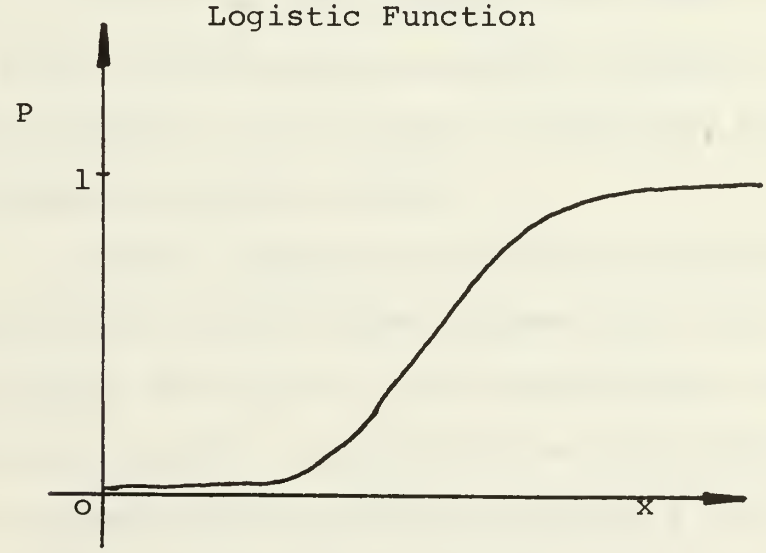
of the logistic function

$$P = (1+e^{-\underline{x}'\underline{\beta}})^{-1} \tag{IV.27}$$

where we immediately assume the multivariate case with $\underline{x}$ being the vector of observations of the independent variables and $\underline{\beta}$ the vector of parameters to be estimated. Note that since we assume a polychotomous response, the model (IV.27) is a reasonable representation of reality. The graph of p vs. x, given for illustrative purposes as Figure IV.4, is a symmetric sigmoid curve with asymtotes at zero and one. This is entirely in keeping with the notion that the observed response is an "either-or" affair; the individual observed either belongs to a particular category or does not.

Figure IV.4

Logistic Function



The logistic function has a long history of use in the field of bioassay along with the integrated normal curve, $f(\mu)$, where

$$f(\mu) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\underline{x}'\beta} \exp(-\tfrac{1}{2}\mu^2)\,d\mu\,, \quad -\infty \leq \underline{x} \leq \infty \qquad \text{(IV.28)}$$

From (IV.28) the probit, P, may be defined as the normal deviate. We shall not dwell on the bioassay applications of the logistic function, but further amplification is provided by Ashton [4]. The impetus for the Walker and Duncan work was an analysis of data arising from a ten-year retrospective study of the medical status of 5,000 individuals. The objective was to determine the predictive relationship between ten variables and the incidence of various degrees of coronary heart disease. In a later application of the method, Talbert [72] was successful in modeling the stagewise development of ova in vitro with a polychotomous response variable. The authors indicate a wide range of additional applications in a number of diverse fields.

A highly interesting connection, demonstrated by Cornfield [19], may be drawn between the form (IV.27) and a particular derivation of the discriminant function by a Bayesian approach. Considering an individual to have a prior probability p of being in group 1 and a (1-p) probability of being in group 2; and if $f_1(\underline{x})$ and $f_2(\underline{x})$

are the likelihood functions for the two groups, then Bayes' Theorem gives the posterior probability of being in group 2 as

$$p_2 = \frac{(1-p)f_2(\underline{x})}{(1-p)f_2(\underline{x})+pf_1(\underline{x})} \tag{IV.29}$$

which yields

$$p_2 = \frac{1}{1+\frac{p}{1-p}\frac{f_1(\underline{x})}{f_2(\underline{x})}} \tag{IV.30}$$

Under the usual multivariate normal assumptions of discriminant analysis $f_1(\underline{x})$ divided by $f_2(\underline{x})$ gives

$$\frac{f_1(\underline{x})}{f_2(\underline{x})} = \exp(-\tfrac{1}{2}[\underline{\mu}_1'\Sigma^{-1}\underline{\mu}_1-\underline{\mu}_2'\Sigma^{-1}\underline{\mu}_2]+\underline{x}'\Sigma^{-1}(\underline{\mu}_1-\underline{\mu}_2)) \tag{IV.31}$$

implying that (IV.30) is of the form

$$p_2 = \frac{1}{1+\exp[-(\alpha+\underline{x}'\underline{\beta})]} \tag{IV.32}$$

which is the same as (IV.27). The major difference between the discriminant function and the logistic function approach, as pointed out by Cox [20], is that the latter is not so highly dependent on the distributional assumption except under some very special cases. He goes on to state that,

> If it is operationally meaningful to consider Prob (y=1) and Prob (y=0) as defined for each fixed $\underline{x}$ and having approximately the form [of a linearized version of (IV.27) taking (p_1/p_2)],

> and if y=1, 0 are the only two possibilities, the approach from [the linear form] seems preferable to that based on the model of two normal x- populations [discriminant analysis], because it assumes less. This applies whether the analysis is made to interpret data, or whether the object is to 'classify' a new individual. [20, p. 64]

Note that the logistic function (IV.27) as given may not be easily transformed to a linear form. Such a nonlinear function is called "intrinsically nonlinear" by Draper and Smith [22]. This has led many authors to use the transform mentioned by Cox, namely

$$\ln \frac{p_2}{p_1} = \alpha + \underline{x}'\underline{\beta} \tag{IV.33}$$

often called the "log odds" or logit. The approach taken by Walker and Duncan is to use instead the function (IV.27) directly via a Taylor Series first-order approximation, then to demonstrate a method whereby the normal equations may be solved by a recursive procedure. This represents a departure from the usual methods of fitting nonlinear models, which are usually based either on iterative schemes to solve the linearized approximation to (IV.23) or steepest descent algorithms that use local gradients as their basic device. The derivation that follows is for the case of a trichotomous response variable, since this is the model we will ultimately utilize, and is based on the derivation given in the original work.

We begin by assuming that with each of our N observations there is associated a response variable which may be captured as

$p_1 = 0$ if the observation does not fall into class 1,

$= 1$ if the observation falls into class 1,

$p_2 = 0$ if the observation does not fall into class 2,

$= 1$ if the observation falls into class 2,

$p_3 = 1 - p_1 - p_2$

$= 0$ if the observation does not fall into class 3,

$= 1$ if the observation falls into class 3.

Essentially we shall fit two functions simultaneously, corresponding to

$$P_1=E(p_1)=f_1=f(\alpha_1,\underline{\beta},\underline{x})=(1+\exp(-\alpha_1-\underline{x}'\underline{\beta}))^{-1} \qquad \text{(IV.34)}$$

and

$$P_1+P_2=E(p_1+p_2)=f_2=f(\alpha_2,\underline{\beta},\underline{x})=(1+\exp(-\alpha_2-\underline{x}'\underline{\beta}))^{-1} \qquad \text{(IV.35)}$$

from which

$$P_3 = 1 - (P_1+P_2) \qquad \text{(IV.36)}$$

can be determined. The assumption made is that

$$P_1 + P_2 \geq P_1 \qquad \text{(IV.37)}$$

which allows us to write f_1 and f_2 with the same "slope," $\underline{\beta}$, and illustrates that the ordinal nature of

the groups is important. That is, group 1 is at a lower level than group 2, both of which are at a lower level than group 3. In the application that follows, this assumption is borne out by noting that a patient requiring Intermediate B level care requires less care than one needing Intermediate A care, both requiring less care than one needing Skilled Nursing care. The model for the n^{th} observation is therefore

$$p_{1n} = f(\alpha_1, \beta, \underline{x}_n) + \epsilon_{1n} \tag{IV.38}$$

$$p_{1n} + p_{2n} = f(\alpha_2, \beta, \underline{x}_n) + \epsilon_{2n}$$

We now take as a linear, first-order approximation to (IV.38) the first two terms of the Taylor Series expansions of each, yielding

$$p_{1n} = f(\bar{\alpha}_1, \bar{\beta}, \underline{x}_n) + \frac{\delta f(\bar{\alpha}_1, \bar{\beta}, \underline{x}_n)\bar{\alpha}_1}{\delta \alpha_1} + \frac{\delta f(\bar{\alpha}_1, \bar{\beta}, \underline{x}_n)\bar{\beta}}{\delta \beta} \tag{IV.39}$$

$$p_{1n} + p_{2n} = f(\bar{\alpha}_2, \bar{\beta}, \underline{x}_n) + \frac{\delta f(\bar{\alpha}_2, \bar{\beta}, \underline{x}_n)\bar{\alpha}_2}{\delta \alpha_2} + \frac{f(\bar{\alpha}_2, \bar{\beta}, \underline{x}_n)\bar{\beta}}{\delta \beta}$$

where $\bar{}_1, \bar{\alpha}_2$, and $\bar{\beta}$ are assumed values of the respective parameters. From (IV.39) we derive so-called "working observations" y^*_{1n} and y^*_{2n} as

$$y^*_{1n} = p_{1n} - f(\bar{\alpha}_1, \bar{\beta}, \underline{x}_n) + \frac{\delta f(\bar{\alpha}_1, \bar{\beta}, \underline{x}_n)\bar{\alpha}_1}{\delta \alpha_1} + \frac{f(\bar{\alpha}_1, \bar{\beta}, \underline{x}_n)\bar{\beta}}{\delta \beta} \tag{IV.40}$$

$$y^*_{2n} = p_{1n} + p_{2n} - (f(\bar{\alpha}_2, \bar{\beta}, \underline{x}_n) + \frac{f(\bar{\alpha}_2, \bar{\beta}, \underline{x}_n)\bar{\alpha}_2}{\delta \alpha_2} + \frac{f(\bar{\alpha}_2, \bar{\beta}, \underline{x}_n)\bar{\beta}}{\delta \beta}$$

The partials may be computed from (IV.27) and found to be

$$\frac{\delta f_1}{\delta \alpha_1} = P_{1n}Q_{1n} \; , \; \frac{\delta f_2}{\delta \alpha_2} = P_{3n}Q_{3n}$$

$$\frac{\delta f_1}{\delta \beta_j} = P_{1n}Q_{1n}x_{jn} \; , \; \frac{\delta f_2}{\delta \beta_j} = P_{3n}Q_{3n}x_{jn} \quad j=1,2,\ldots,s \tag{IV.41}$$

where

s = the number of variables plus two,

P_{1n} and P_{3n} are as given by (IV.34) and (IV.36) respectively, and

Q_{1n} and Q_{3n} are one minus P_{1n} and P_{3n}, respectively.

It should be obvious at this point that since the individual equations of the model (IV.38) are interrelated, some sort of weighting corresponding to a correlation must be given to the data matrix X. It can be shown that such a weight matrix will be the inverse of the diagonal matrix of variance and covariances of the errors, whose elements are the 2 x 2 matrices:

$$w_n = V^{-1}(\underline{\epsilon}_n) = \frac{1}{\hat{P}_{2n}} \begin{bmatrix} \dfrac{\hat{Q}_{3n}}{\hat{P}_{1n}} & -1 \\ -1 & \dfrac{\hat{Q}_{1n}}{\hat{P}_{3n}} \end{bmatrix} . \tag{IV.42}$$

We may now specify the model in terms of the working observations $\underline{y}^*$ as

$$\underline{y}^* = AX\underline{\theta} + \underline{\epsilon} \tag{IV.43}$$

where

$$\underline{y}^* = \begin{bmatrix} y^*_{11} \\ y^*_{21} \\ \cdot \\ \cdot \\ \cdot \\ y^*_{1N} \\ y^*_{2N} \end{bmatrix} \tag{IV.44}$$

$$\underline{\theta} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \beta_1 \\ \cdot \\ \cdot \\ \cdot \\ \beta_s \end{bmatrix} \tag{IV.45}$$

$$X = \begin{bmatrix} 1 & 0 & x_{11} & x_{12} & \cdots & x_{1s} \\ 0 & 1 & x_{11} & x_{12} & \cdots & x_{1s} \\ & & & \cdot & & \\ & & & \cdot & & \\ & & & \cdot & & \\ 1 & 0 & x_{N1} & x_{N2} & \cdots & x_{Ns} \\ 0 & 1 & x_{N1} & x_{N2} & \cdots & x_{Ns} \end{bmatrix} \tag{IV.46}$$

$$\underline{\epsilon} = \begin{bmatrix} \epsilon_{11} \\ \epsilon_{21} \\ \vdots \\ \epsilon_{1N} \\ \epsilon_{2N} \end{bmatrix} \tag{IV.47}$$

$$A = \begin{bmatrix} A_1 & & 0 \\ & A_2 & \\ & & \ddots \\ 0 & & A_n \end{bmatrix} \tag{IV.48}$$

where

$$A_n = \begin{bmatrix} P_{1n}Q_{1n} & 0 \\ 0 & P_{3n}Q_{3n} \end{bmatrix}, \; n=1,2,\ldots,N \tag{IV.49}$$

in accordance with the form of (IV.40). The analogous equation to (IV.23) is therefore

$$\underline{\hat{\theta}} = (X'AWAX)^{-1}AW'W\underline{y}^*, \tag{IV.50}$$

and we may also write

$$V(\underline{\hat{\theta}}) = (X'AWAX)^{-1} \tag{IV.51}$$

The derivation to this point has brought us to the form (IV.50) which, as was mentioned earlier, might be solved using an iterative algorithm. Walker and Duncan, however, suggest an alternative procedure based on a recursive scheme that updates estimates of $\underline{b}_n = \underline{\hat{\theta}}_n$ and V_n at the consideration of each observation. Without proof, the appropriate relationships are

$$V_n = V_{n-1} - V_{n-1}\underline{x}_n A_n D_n^{-1} A_n \underline{x}_n' V_{n-1} \tag{IV.52}$$

where

$$D_n = A_n \underline{x}_n' V_{n-1} \underline{x}_n A_n + W_n^{-1} \tag{IV.53}$$

and

$$\underline{b}_n = \underline{b}_{n-1} + V_{n-1}\underline{x}_n A_n D_n \begin{bmatrix} p_{1n} - \hat{P}_{1n} \\ p_{1n} + p_{2n} - (\hat{P}_{1n} + \hat{P}_{2n}) \end{bmatrix} \tag{IV.54}$$

and where

$\hat{P}_{1n}$ and $\hat{P}_{2n}$ are (IV.27) appropriately evaluated with the best current estimates of $\underset{\sim}{\beta}$ being used.

This essentially completes the derivation save for some computational considerations.

First of all, it can be seen that the scope of one iteration in this method encompasses the ultimate use of all available data, with the estimates of $\underset{\sim}{\beta}$ hopefully stabilizing within its course. A convergence check based on the final $\underline{b}$ and the initial guess $\underline{b}_o$ is made after each such iteration, a useful form being

$$\left| \frac{b_i - b_{oi}}{b_i} \right| \leq \delta \qquad i=1,2,\ldots,s \tag{IV.55}$$

where δ is a tolerance value, say 10^{-4}. Additionally, the authors demonstrate that at any stage of an iteration the effect of the initial guesses of $\underline{b}$ and V may be removed by application of the following, where k denotes

the observation at which the correction takes place and starred quantities indicate the current values.

$$V_k = (V_k^{*-1} - V_o^{-1})^{-1}$$
$$\underline{b}_k = V_k(V_k^{*-1}\underline{b}_k^{*} - V_o^{-1}\underline{b}_o) \qquad \text{(IV.56)}$$

As a final computational feature, an initial centering of the data by subtraction of the arithmetic mean of each variable assists in obtaining orthogonality between the estimates of α_1 and α_2. Presumably, if strong colinearity among the variables is suspected, methods which yield orthogonal columns of the data matrix X could be applied; these were not attempted in this study.

Upon fitting the model (IV.38) by obtaining estimates α_1^*, α_2^*, and $\underline{\beta}^*$ we would naturally desire to have a classification rule and some measures of the effectiveness of the model which are either self-contained or rule-dependent. Towards the end of deriving the rule, note that the following posterior probabilities of an individual $\underline{x}_n$ being in each of the three groups may be computed after fitting the model:

$$p_{1n}^* = (1 + \exp(-\alpha_1^* - \underline{x}_n'\underline{\beta}^*))^{-1} \qquad \text{(IV.57)}$$

$$p_{2n}^* = (1+\exp(-\alpha_2^* - \underline{x}_n'\underline{\beta}^*))^{-1} - (1+\exp(-\alpha_1^* - \underline{x}_n'\underline{\beta}^*))^{-1} \qquad \text{(IV.58)}$$

$$p^*_{3n} = 1 - \exp(-\alpha_2^* - \underline{x}_n'\underline{\beta}^*))^{-1} \qquad \text{(IV.59)}$$

We therefore propose the following simple classification rule:

For an individual represented by the vector of observations $\underline{x}_n$, classify as being in group i if

$$p^*_{in} > p^*_{jn}, \text{ for all } j \neq i,\ j=1,2,3 \qquad \text{(IV.60)}$$

where p^*_{in} are given by (IV.57), (IV.58), (IV.59).

In order to check the merit of the classification rule, a classification analysis procedure suggested by Parker [52,53] may be utilized. This essentially involves the comparison of the prior and posterior classifications, individual by individual, agreement constituting a "hit" while disagreement implies a "miss." Note that the prior classification is assumed to be

$$\underset{i}{\text{MAX}}\ p_{in}, \qquad i=1,2,3,$$

and represents professional judgment as to a patient's appropriate level of care; and the posterior classification is

$$\underset{i}{\text{MAX}}\ p^*_{in}, \qquad i=1,2,3,$$

and represents the model's prediction of a patient's level of care. As a means of displaying the dispersion of misses, a classification matrix can be formed by entering in the appropriate cells the number of individuals with the given comparison of prior and posterior classifications. Additional information available

includes both a measure of the prediction and recognition powers of the model in terms of the prediction and recognition rates, respectively. The prediction rate gives the percentage of patients with prior classification i among all patients assigned by the model to classification i. The recognition rate, on the other hand, is the percentage of patients with prior classification i which was asserted by the model to have the same posterior classification. The form of the classification matrix is shown in Figure IV.5.

Figure IV.5

Classification Matrix as Suggested by Parker

		Posterior Classification 1	2	3	Recognition Rate:
Prior Classification	1	a	b	c	a/a+b+c
	2	d	e	f	e/d+e+f
	3	g	h	i	i/g+h+i
Prediction Rate:		a/a+d+g	e/b+e+h	i/c+f+i	

As an added measure, the usual type of analysis of variance (ANOVA) may be performed. We based our calculations on the linear approximations to the functions $f(\alpha_1^*, \underline{x}_n, \underline{\beta}^*)$ and $f(\alpha_2^*, \underline{x}_n, \underline{\beta}^*)$ provided by the working

observations (IV.40) evaluated at $\alpha_1^*, \alpha_2^*, \beta^*$. We denote the (2N x 1) vector of such approximations as $\underline{y}^*$ and the (2N x 1) vector of errors

$$\underline{E} = (\underline{y} - \hat{\underline{y}}) \tag{IV.61}$$

where

$\hat{\underline{y}}$ is simply the (2N x 1) vector of pairs consisting of the logistic functions evaluated at α_1^*, β^*, and $\underline{x}_n$ and α_2^*, β^*, and $\underline{x}_n$.

Recalling that p is the number of variables under consideration, we write the ANOVA in the following format:

Figure IV.6

ANOVA for Logistic Model

ANOVA (Uncorrected for Mean)

Source	Degrees of Freedom	Sum of Squares	Mean Square	F Ratio
Regression	p+2	$\beta^{*\prime} X W \underline{y}^*$	SSR/df	MSR/MSE
Error	2N - p - 2	$\underline{E}' W \underline{E}$	SSE/df	
Total	2N-1	$\beta^{*\prime} X W \underline{y}^* + \underline{E}' W \underline{E}$		

Note that the ANOVA has been derived without a correction for the mean of $\underline{y}^*$ because the concept of a mean element value is obscured by the pair-wise nature of the elements themselves. Nevertheless, the information available from Figure IV.6 provides a good indication of explained variation. Observe, however, that due to the fact that a

linear approximation to the logistic is being fitted by our model, the vector $\underline{y}^*$ is dependent upon the estimates derived and will not, in general, be invariant with respect to alterations in the number of variables, p, for example. This will have implications in computing the regression and total sums of squares, since in the ordinary linear regression case the quantity used is $\underline{y}$, which is invariant.

It is perhaps wise at this point to pause and emphasize a few general points concerning the use of regression-type models in classificatory applications. The first such item was mentioned earlier but bears repeating: the relationships we seek between the independent variables (regressors) and the dependent variable (response) are of a predictive nature versus a causative one. In general, the functional specifications of each of these relationships will differ, and this is often a source of confusion for the unwary user of the results. In terms of our attempt to classify long-term care patients this caveat implies that the outcomes of the study should not be viewed as relating those factors deemed to be of consequence as <u>directly</u> causing a patient, of necessity, to be placed in a particular level of care. Rather, the preferred interpretation is that for the data available the quantitatively salient factors were

able to explain a significant portion of the variation caused by attempting to obtain congruence between a very large (although finite) number of possible vectors denoting patient status and three levels of care. It is therefore assumed in all such studies that the state of the world with regard to long-term care patient status is fairly regular, thus allowing the derived model to be used for predictive purposes, with the added caution given by Draper and Smith [22, p. 241] to, "...restrict prediction to the region of x-space from which the original data were obtained."

An additional consideration in this regard concerns that portion of the variation mentioned above which the model cannot explain. A more general specification of (IV.38) might be as follows:

$$p_{1n} = g(\alpha_1, \underline{\beta}, \underline{x}_n; \theta_1, \theta_2, \ldots, \theta_k; \underline{m}) \qquad \text{(IV.62)}$$

$$p_{1n} + p_{2n} = g(\alpha_2, \underline{\beta}, \underline{x}_n; \theta_1, \theta_2, \ldots, \theta_k; \underline{m})$$

where parameters previously used are as before, but g denotes a general functional form which is also parametric on θ_i, i=2,...,k which are unknown to us, and $\underline{m}$ is a vector of measurement errors in the regressors. A model such as (IV.62) can be used to illustrate the assumption that the unexplained variation is due to factors which are obscured to the researcher and to measurement error. The model applied is based upon such

an assumption. Blalock [9] summarizes and extends these concepts in a concise fashion by noting that a form similar to (IV.62), but without $\underline{m}$, is usually acceptable in cases where the relative magnitude of error due to unknown factors is assumed to be greater than that attributable to measurement error. He goes on to point out that such a form is also a reasonable specification for nonexperimentally derived data; that is, data which are obtained without the benefit of experimental design to control for effects. Methods for isolating the effects of measurement errors and errors due to unknown factors (when both are assumed to be present) are relatively undeveloped.

IV.5 The Variable Subset Selection Problem

Near the outset of the chapter the question of feature extraction was emphasized as being an integral part of the pattern recognition process. Mention was also made of the problem involving the identification of the optimal set of features of size n, the so-called optimal subset selection problem. Some additional comments on each of these are in order.

After having fitted the model using all of the appropriate and available data at our disposal, it becomes necessary to reformulate our procedure somewhat so

that the classification function may be placed in a useful framework for the ultimate consumer. Usually this reformulation is characterized by a reduction in the number of original variables used to effect the model fit. There are two main reasons for this. First, if the marginal cost of obtaining each additional piece of data is relatively high it behooves us to present as low-cost a solution as can be obtained. The second consideration is that it is entirely possible that certain of the original variables might actually be obscuring the predictive power of the model by interacting in a negative way with other variables. The elimination of such variables could therefore lead to an improved model if the prediction rate is the criterion used to choose between alternatives.

Theoretically, at least, we may postulate the existence of a subset of the variables of size n which would yield the best fit (in terms of some criterion) among all other subsets of size n. In addition, if we assume that the marginal cost of obtaining each variable subsequent to the first few is constant, then n could be interpreted to be the maximum number of variables that could be obtained within budgetary limits. The problem posed by the attempt to find such subsets is called an optimal subset selection problem, the criterion in the

more general situation usually being one determined from information theoretic concepts.

For the case of ordinary multiple linear regression, Boyce, Farhi, and Weischedel [11] have proposed an enumerative algorithm which determines the variables constituting an optimal subset of size n based on the criterion of percentage of variance explained. This essentially combinatorial problem is solved by establishing bounds to implicitly evaluate certain combinations of variables which are dominated by others. The search is bounded and exhaustive. Further, the authors claim that the method is superior to the feature selection methods posed earlier in the chapter due to its ability to find the global optimum versus local optima. Although attractive, the algorithm does not appear to be readily applicable to the non-linear regression problem, primarily because of the dependence of the regression and total sums of squares on the estimated parameters via the vector $\underline{y}^*$. When considering problems of size n, for example, the total sum of squares would not be invariant, but would depend upon the fit itself and thus would lead to difficulties in establishing the necessary bounding relationships.

In the absence of such refined methodology, we have attempted to establish a reasonable list of

candidate subsets for examination both by following the recommendations of Fisher, et al. [23] and invoking the technique of backward elimination from ordinary MLR. By doing so we will not, in all likelihood, find the optimal subsets of whatever sizes are chosen as being within reason, but as will be shown below the criteria do not appear to be highly sensitive.

IV.6 Results

We turn now to the specific application of the methodology to the problem of patient classification. Recalling that we initially seek a recognition of patterns in the data as they relate to the response variable, and then to select features in order to devise an operational classification system, we proceed as follows.

Making use of the available data from the Collaborative Patient Assessment Instrument (CPAI) which was discussed in Chapter III, the following modifications were made. First, the original sample of 623 patient records was reduced by 295 through elimination of the data obtained by the Harvard research group. The rationale for this action was twofold. Discussions with researchers in the study which produced the CPAI indicated that the criteria used by the experts in judging the appropriate level of care for the eliminated data were

different from those used by the other two groups. Parker [53] noted a difference between the Harvard data and the data from the other two groups. After application of discriminant analysis to the CPAI data with judgment of "appropriate level of care" categorized by research groups as the dependent variable, a high degree of group variability was found between the research groups. This phenomenon was not, however, as pronounced between the Hopkins and HANYS data.

A further reduction of 22 individuals was made to eliminate patients deemed to have an appropriate care level of "chronic care" or patients with incomplete records. Finally, on the recommendation of professional persons the record sets of patients with an appropriate care level of "Domiciliary" and "Intermediate B" were merged since these patients are likely to be quite similar with respect to medical, functional, and psychosocial status, other factors intervening in their ultimate level of care assignment. The final sample used in the analysis is depicted in Table IV.1

Turning our attention to the set of variables to be used, recall that Table III.1 of Chapter III listed some 60 items included in the original CPAI, falling into the broad categories of socio-demographic data, functioning status items, impairments, and medical status items.

Table IV.1

Derived Sample for Classification Study

	Appropriate Level of Care Int. B	Int. A	SNF	Total
Sample Size	66	110	130	306

It will also be remembered that we are seeking a post-assessment patient classification system whose purpose is to assist in predicting the demand for the level of nursing services in the long-term care setting. Of equal import is the potential value of the classification process for the placement of patients into an appropriate level of care, noting that other factors will also, of necessity, be considered. Since the main thrust of both purposes is related to what may broadly be termed a patient's health status, it is obvious that certain of the CPAI items do not and should not be considered in our system. Such variables fall mainly under the category of socio-demographics and as such were eliminated from the study. Other variables eliminated were those found to have such a low incidence of variation they could be considered to be constant over all data, and those items that were missing in a significant proportion of the records. The results of other studies cited in Chapter II tend to indicate, with the exception of

the socio-demographic variables, that the variables eliminated should not be of major predictive value. Table IV.2 lists those variables that were ultimately included. In terms of the broad categorization used earlier, we can see that items 1-15 are functioning status items, 16-25 impairment items, and 26-37 medical status items, thus representing a desirable set of variables for the stated purposes of our study.

The method of Walker and Duncan described in Section 4 was coded in FORTRAN IV for execution on the DEC System 10. Using the appropriate level of care as the dependent variable with its three possible values of Intermediate B, Intermediate A, and Skilled Nursing Facility, a run was made using all 37 variables listed in Table IV.2. After completion of the calculations, an ANOVA and classification analysis were performed. In the classification analysis, "assigned level of care" refers to the assignment made by the model, while "appropriate level of care" refers to the judgments of professionals as to the level of care the patient should be receiving. Table IV.3a shows the resulting estimates of α_1, α_2, and β by variable number, as well as the ratio of β_i^* to s_i, where s_i is the sample standard deviation of the estimate. Table IV.3b gives the ANOVA, and Table IV.3c the classification analysis. As a further

Table IV.2

Variables Selected for Classification Study

1.	Mobility Level	19.	Fractures and Dislocations
2.	Transferring	20.	Joint Motion (Upper)
3.	Walking	21.	Joint Motion (Lower)
4.	Wheeling	22.	Joint Pain and Swelling
5.	Stair Climbing	23.	Missing Limbs (Lower)
6.	Bathing	24.	Paralysis/Paresis
7.	Dressing	25.	Dentition
8.	Eating/Feeding	26.	Cigarette Smoking
9.	Toileting	27.	Anemia
10.	Bowel Function	28.	Angina and/or M.I.
11.	Bladder Function	29.	Arthritis
12.	Orientation	30.	Cardiac Arrhythmias
13.	Communication of Needs	31.	Congestive Heart Failure
14.	Behavior Pattern (Freq.)	32.	Diabetes Mellitus
15.	Behavior Pattern (Type)	33.	Hypertension, Essential (Incl. Malignant Hypertension)
16.	Sight Impairment	34.	Malignancy
17.	Hearing Impairment	35.	Mental Illness
18.	Speech Impairment	36.	Neurological Disorder(s)
		37.	Chronic Respiratory Disease

amplification of Table IV.3, note that the F-test for evidence of overall regression is highly significant with $p < .001$. Also, the overall recognition rate was 67% while the overall prediction rate was about 70%. Notice, too, that all but about 5% of the misses occurred one step off the main diagonal, with a noticeable skew towards a higher level of assignment. The assumed model obviously had the most difficulty in predicting placement

Table IV.3a

Results of Application of Walker and Duncan Method for 37 Variables

Variable	β_i^*	β_i^*/s_i	Variable	β_i^*	β_i^*/s_i
α_1	2.413	--	17	0.125	0.885
α_2	5.200	--	18	-0.153	-0.884
1	-0.310	-3.100	19	-0.192	-1.356
2	-0.127	-0.693	20	-0.238	-0.840
3	0.274	0.584	21	-0.019	-0.079
4	0.120	1.198	22	0.038	0.170
5	-0.441	-4.413	23	0.223	1.287
6	-0.439	-2.197	24	-0.133	-0.664
7	-0.322	-1.609	25	-0.170	-1.200
8	-0.470	-2.712	26	0.604	2.134
9	0.358	1.791	27	0.723	1.417
10	-0.114	-0.805	28	-0.345	-0.629
11	-0.008	-0.083	29	-0.297	-0.939
12	-0.197	-0.657	30	0.472	0.776
13	-0.048	-0.127	31	-0.104	-0.259
14	-0.379	-1.012	32	-0.424	-1.134
15	0.618	2.185	33	0.307	0.852
16	-0.204	-1.018	34	-0.279	-0.595
			35	-1.810	-2.419
			36	-0.370	-1.169
			37	-1.095	-2.283

Table IV.3b

ANOVA for Application with 37 Variables

Source	df	SS	MS	F
Regression	36	3774.02	104.83	44.5
Error	269	633.74	2.36	---
Total	305	4407.76	---	---

Table IV.3c

Classification Matrix for Application with 37 Variables

		Assigned Level of Care				
		Int. B	Int. A	SNF	Total	Recognition Rate
Appropriate Level of Care	Int. B	41	21	4	66	.621
	Int. A	10	69	31	110	.627
	SNF	1	29	100	130	.767
	Total	52	119	135	306	.672
Prediction Rate		.778	.580	.740	.700	

of "Intermediate A" patients. Some further comments on these results will be made later.

Having seen the results made available by fitting the logistic function model for all variables under consideration, we now turn to the subset selection problem. Using both the intuitive and statistical rationales explained earlier, a total of six subsets of the original 37 variables were derived and are listed in Table IV.4. The first two of these sets were formed from results reported by Parker and Boyd [53] in the application of discriminant and cluster analysis, respectively. Variables which were included in those studies but not in the present one were naturally excluded, and some additions were made to the second subset based on the first. The next two sets were determined from an

Table IV.4

Subsets of the Original Variables Chosen for Analysis

Subset I (Parker and Boyd Discriminant Analysis with modifications)

1. Mobility Level	7. Communication
2. Wheeling	8. Behavior (Type)
3. Bathing	9. Fractures and Dislocations
4. Dressing	10. Paralysis/Paresis
5. Toileting	11. Dentition
6. Orientation	12. Neurological Disorder(s)

Subset II (Parker and Boyd Cluster Analysis with modifications)

1. Mobility Level	7. Bowel Function
2. Wheeling	8. Communication
3. Bathing	9. Anemia
4. Dressing	10. Mental Illness
5. Eating/Feeding	11. Neurological Disorder(s)
6. Toileting	12. Chronic Respiratory Disease

Subset III (Cluster Analysis One)

1. Mobility Level	7. Toileting
2. Transferring	8. Bowel Function
3. Walking	9. Bladder Function
4. Wheeling	10. Orientation
5. Bathing	11. Communication
6. Eating/Feeding	12. Speech Impairment

Subset IV (Cluster Analysis Two)

1. Transferring	7. Toileting
2. Walking	8. Bowel Function
3. Wheeling	9. Bladder Function
4. Bathing	10. Orientation
5. Dressing	11. Communication
6. Eating/Feeding	12. Mental Illness

Subset V (Backward Elimination, $\alpha=.10$)

1. Mobility Level	7. Toileting
2. Walking	8. Behavior (Type)
3. Stair Climbing	9. Cigarettes
4. Bathing	10. Anemia
5. Dressing	11. Mental Illness
6. Eating/Feeding	12. Chronic Respiratory Disease

Subset VI (Backward Elimination, $\alpha=.05$)

1. Mobility Level	6. Behavior (Type)
2. Stair Climbing	7. Cigarettes
3. Bathing	8. Mental Illness
4. Eating/Feeding	9. Chronic Respiratory Disease
5. Toileting	

application of Parker and Boyd's hierarchical centroid clustering algorithm to combined samples of Intermediate and Skilled care patients, the variables being chosen by the B/T ratio criterion. The final two subsets were formed after application of a t-test of significance of the estimated regression coefficients in the 37-variable problem, one being taken at $\alpha \doteq .10$, the other at $\alpha \doteq .05$. The appropriate test statistic is:

$$t = \frac{\beta_i^* - \beta_i^o}{s_i} \qquad \text{(IV.63)}$$

for which we test

$$H_0 : \beta_i^o = 0$$

$$H_1 : \beta_i^o \neq 0 \quad .$$

Using the two-tailed t-test, under H_0, the t-statistic values are given as β_i^*/s_i in Table IV.3a. Notice that all but the final subset contain 12 variables, a fact which was arrived at both by chance and design. The decision was made based on the fact that each subset originally contained about 12 variables, and the reduction in data collection effort by two-thirds obtained by choosing 12 variables seemed reasonable. The 9-variable subset represents an attempt to further reduce the number of variables based on the encouraging result obtained by use of subset V.

For the sake of compactness, the results of fitting the model for each of the subsets are given in Table IV.5. Results of each ANOVA are summarized in Table IV.6. The ANOVA for each of the subsets shows evidence of overall regression to be strong, with $p < .001$ in each case. The ratio SSR/SST provides a gross indication of an upper bound for R^2 since, in general,

$$R^2 = \frac{\text{SSR - (Correction for Mean)}}{\text{SST - (Correction for Mean)}} \tag{IV.64}$$

and since SSR $\leq$ SST, then

$$R_u^2 = \frac{\text{SSR}}{\text{SST}} \geq R^2 \ , \tag{IV.65}$$

If we were to choose R_u^2 as the criterion for deciding which subset was best, subsets I, V, and VI might be worthy of further consideration, although all six yield approximately equal R_u^2. Such a measure is not, however, in keeping with the spirit of the analysis; that is, derivation of a method which has reasonably high predictive validity for classification purposes. We therefore present the classification matrices as Tables IV.7 through IV.12 to assist us in reaching a decision. To facilitate the decision process, the following measures of performance may be abstracted from the classification matrices for presentation in Table IV.13:

Table IV.5

Estimated β_i from Logistic Model for the Six Subsets of Table IV.4

Variable	SUBSETS I	II	III	IV	V	VI
Mobility Level	-0.372	-0.404	-0.360	--	-0.294	-0.301
Transferring	--	--	-0.224	-0.304	--	--
Walking	--	--	0.051	-0.006	0.126	--
Wheeling	0.071	0.105	0.109	0.098	--	--
Stair Climbing	--	--	--	--	-0.433	-0.406
Bathing	-0.664	-0.630	-0.672	-0.707	-0.591	-0.638
Dressing	-0.493	-0.357	--	-0.427	-0.368	--
Eating/Feeding	--	-0.494	-0.534	-0.488	-0.557	-0.665
Toileting	0.357	0.491	0.362	0.432	0.428	0.346
Bowel Function	--	-0.116	-0.083	-0.097	--	--
Bladder Fcn.	--	--	0.021	0.028	--	--
Orientation	-0.275	--	-0.138	-0.071	--	--
Communication	-0.285	-0.287	-0.315	-0.286	--	--
Behavior (Type)	0.204	--	--	--	0.283	0.272
Speech Impair.	--	--	-0.008	--	--	--
Fractures	-0.211	--	--	--	--	--
Paralysis	-0.094	--	--	--	--	--
Dentition	-0.202	--	--	--	--	--
Cigarettes	--	--	--	--	0.644	0.698
Anemia	--	0.560	--	--	0.628	--
Mental Illness	--	-1.376	--	-1.000	-1.812	-1.821
Neurol. Dis.	-0.131	-0.243	--	--	--	--
Chron. Res. Dis.	--	-0.868	--	--	-1.007	-1.017
α_1^*	1.563	1.191	0.932	0.829	1.817	1.801
α_2^*	4.008	3.706	3.354	3.222	4.506	4.431

Table IV.6

ANOVA Summary for the Six Subsets of Table IV.4

Quantity	SUBSETS I	II	III	IV	V	VI
MSR	246.86	205.98	216.79	217.67	262.33	353.57
MSE	2.06	2.52	2.29	2.18	2.32	2.24
F	119.75	81.64	94.71	100.02	112.91	157.97
df	11,294	11,294	11,294	11,294	11,294	8,297
SSR	2715.5	2265.76	2384.71	2394.42	2885.64	2828.54
SST	3321.6	3007.51	3057.71	3034.28	3568.69	3493.31
SSR/SST	.83	.75	.78	.79	.81	.81

1. Overall prediction rate,
2. Overall recognition rate,
3. Percentage of misses $>$ 1 step off main diagonal.

With the information made available by these tables, certain conclusions may be reached concerning a choice of subset. On all but the third of the performance measures proposed, the fifth subset is shown superior to all other subsets examined as well as the original 37-variable problem. This last point tends to lend credence to the hypothesis of negative interaction among some of the variables which appears as error due to the specification of non-interaction in the original model. The non-interactive logistic function model is probably a better representation of reality for subsets V and VI. Subset VI performed about as well as the original model in terms of the first two measures. The original model does appear to give a higher percentage of "near misses" than the reduced forms, but certainly not so significant an amount as to outweigh its additional workload. Finally, it can be observed that all seven attempts at fitting the model tended to give a slight preponderance of misses wherein a patient was deemed to be properly placed in a higher level of care than prior judgment indicated. This can be seen by examining the totals for the assigned

Table IV.7

Classification Matrix for Subset I

Appropriate Level of Care		Assigned Level of Care: Int. B	Int. A	SNF	Total	Recognition Rate
	Int. B	33	26	7	66	.500
	Int. A	14	63	33	110	.573
	SNF	4	31	95	130	.730
	Total	51	120	135		
Prediction Rate		.647	.525	.703		

Table IV.8

Classification Matrix for Subset II

Appropriate Level of Care		Assigned Level of Care: Int. B	Int. A	SNF	Total	Recognition Rate
	Int. B	41	19	6	66	.621
	Int. A	12	67	31	110	.609
	SNF	0	32	98	130	.754
	Total	53	118	135		
Prediction Rate		.774	.568	.726		

Table IV.9

Classification Matrix for Subset III

		Assigned Level of Care: Int. B	Int. A	SNF	Total	Recognition Rate
Appropriate Level of Care	Int. B	33	27	6	66	.500
	Int. A	9	67	34	110	.609
	SNF	1	31	98	130	.754
	Total	43	125	138		
Prediction Rate		.767	.536	.710		

Table IV.10

Classification Matrix for Subset IV

		Assigned Level of Care: Int. B	Int. A	SNF	Total	Recognition Rate
Appropriate Level of Care	Int. B	34	25	7	66	.515
	Int. A	11	63	36	110	.573
	SNF	3	30	97	130	.746
	Total	48	118	140		
Prediction Rate		.708	.534	.693		

Table IV.11

Classification Matrix for Subset V

		Assigned Level of Care				
		Int. B	Int. A	SNF	Total	Recognition Rate
Appropriate Level of Care	Int. B	44	17	5	66	.667
	Int. A	9	67	34	110	.609
	SNF	2	23	105	130	.808
	Total	55	107	144		
Prediction Rate		.800	.626	.729		

Table IV.12

Classification Matrix for Subset VI

		Assigned Level of Care				
		Int. B	Int. A	SNF	Total	Recognition Rate
Appropriate Level of Care	Int. B	43	17	6	66	.652
	Int. A	9	66	35	110	.600
	SNF	2	27	101	130	.777
	Total	54	110	142		
Prediction Rate		.796	.600	.711		

Table IV.13

Summary of Performance Measures for the Six Subsets

Measure	I	II	III	IV	V	VI
(1)	.624	.689	.671	.645	.718	.702
(2)	.598	.661	.621	.611	.695	.676
(3)	9.6	6.0	6.5	8.9	7.8	8.3

[illegible]

[illegible]

[illegible]

[illegible]

levels of care as compared with the totals for the appropriate levels of care. The net effect of this characteristic of the system is to make the classification a bit conservative when taken over the patient population of a facility. But since the ultimate use of this scheme is in predicting nursing demands, the nature of the errors made could lead to slight over-staffing, a situation perhaps more tolerable from the patient viewpoint than the reverse. This implies, of course, that the assumed cost of misclassification is equal whether the misclassification is to a higher or lower level of care.

It is worthy of mention that since further discussions with CPAI research personnel indicated the consideration of factors other than health status in adjudicating appropriate level of care (e.g., insurance arrangements, relatives at home, etc.), some attempt was made to analyze the records of those patients who missed on several of the runs. Only a general impression that the health status might be closer to the assigned versus appropriate level of care estimates could be obtained, however, due to unavailability of professional persons to reassess records in light of the current study goals.

We conclude, therefore, by noting that subsets V and VI are the groupings of choice for consideration in

an operational patient classification system. Recall that no claim is made that these are <u>the</u> optimal subsets of size 12 and 9, respectively, in terms of any criterion; rather, they represent reasonable approximations due to an apparent insensitivity of the model among both intuitively and statistically derived subsets. What now remains is the translation of the fruits of the analysis to an instrument engineered for use by professional nurses in a typical long-term care facility.

Factors to be considered in the design of the patient classification form include the assurance that the same assessment scales will be used as in the CPAI data, that easily followed directions for use will be available, that a clear format be developed, and that a simple way to reproduce the pattern recognition function served by the logistic model be provided. The first of these items may be provided by including a table of assessment scores with the form, these being abstracted from the CPAI User's Manual [35]. The next two are implementation problems for which we propose a possible example. The last item is dealt with by use of easily followed formulae requiring addition, subtraction, multiplication and three table look-ups in a table provided. In this last regard, the sensitivity of the model to a rounding of its estimated coefficients to one decimal

place was tested for subsets V and VI with a degradation of no more than 2% being observed in either the prediction or recognition rates. Hence, this more easily used format is recommended.

Figure IV.7 below illustrates the proposed patient classification form based on subset V. If it is desired to use subset VI instead, appropriate substitutions may be made using the estimates provided in Table IV.5. Note that the entire range of possible values of the logistic function exponent is represented in the "table of values."

As a final note, the classification form of Figure IV.7 may be further modified, if space permits, to include the display of assessment and classification over time by duplicating the fourth and fifth columns with an appropriate notation for the date.

IV.7 Final Remarks

The analysis of this chapter has led us through the general pattern recognition problem to a specific model from which the essentials of the recognition problem could be drawn, and thence to subset selection and the presentation of a proposed patient classification instrument based on medical and functioning status considerations only. The aggregate numbers of patients

Figure IV.7

Patient Classification Form

Name ______________________ I.D.# ____________ Room _________

Directions: Assess the patient with respect to the items below in accordance with the attached rating scale. Multiply the score obtained by the factor indicated and sum the results. Find Total 1 and Total 2 as shown, then consult the enclosed table to obtain values for Class B, Class A, and Class S. The maximum of these three numbers yields the Estimated Classification (Int. B, Int. A, SNF)

Item #	Description	Factor	Assessment	Factor x Assessment
1	Mobility	-0.3		
2	Walking	0.1		
3	Stair Climbing	-0.4		
4	Bathing	-0.6		
5	Dressing	-0.4		
6	Eating/Feeding	-0.6		
7	Toileting	0.4		
8	Behavior (Type)	0.3		
9	Cigarettes	0.6		
10	Anemia	0.6		
11	Mental Illness	-1.8		
12	Chron.Res.Dis.	-1.0		

Total = ____________

Total 1 = Total + 1.8 = ____________

Total 2 = Total + 4.5 = ____________

Class B = Value of Total 1 = ____________

Class A = Value of Total 2 - Value of Total 1 = ____________

Class S = 1 - Value of Total 2 = ____________

Class = ____________

Figure IV.7, continued

Assessment Rating Scales

(Note: HA means Human Assistance, MA means Mechanical Assistance)
[from: Patient Classification for Long-Term Care: User's Manual]

Item	Description	Score	Interpretation
1	Mobility Level	0	Gets outside without either MA or HA
		1	Gets outside with MA but without HA
	.	2	Gets outside with HA, with or without MA
		3	Confined to institution, moves about without HA or MA
		4	Confined to institution and moves about with MA but without HA
		5	Confined to institution, moves about with HA, with or without MA
		6	Confined to bed or chair
2	Walking	0	Walks without MA or HA
		1	Walks with MA but without HA
		2	Walks with HA but withoug MA
		3	Walks with HA and MA
		4	Does not walk (bed or chair)
		5	Does not walk (bedfast)
3	Stair Climbing	0	Goes up and down flight of stairs without HA or MA
		1	Goes with MA but not HA
		2	Goes with HA but not MA
		3	Goes with MA and HA
		4	Does not perform
4	Bathing	0	Bathes self without assistance
		1	Bathes self with MA only
		2	Bathes self with HA, with or without MA
		3	Does not bathe self
5	Dressing	0	Dresses self without assistance
		1	Dresses self with MA only
		2	Dresses self with HA, with or without MA
		3	Does not dress self
6	Eating/Feeding	0	Feeds self without assistance
		1	Feeds self with MA only
		2	Feeds self with HA, with or without MA
		3	Does not feed self, is fed
		4	Does not feed self, tube fed
		5	Does not feed self; parenteral administration of fluids

Figure IV.7, continued

Assessment Rating Scales, Continued

Item	Description	Score	Interpretation
7	Toileting	0	Uses toilet room without assistance
		1	Uses toilet room with MA only
		2	Uses toilet room with HA, with or without MA
		3	Does not use toilet room
8	Behavior (Type)	0	Appropriate
		1	Inappropriate, wandering or passive
		2	Inappropriate, abusive or aggresive
		3	Inappropriate, other
9	Cigarette Smoking	0	Not current smoking
		1	Smoking less than 15 cigarettes/day
		2	15-24 cigarettes/day
		3	25 or more/day
10	Anemia	0	Not present
		1	Present (Aplastic, pernicious, folic acid, sickle cell, or combinations)
11	Mental Illness	0	Not present
		1	Present
12	Chronic Respiratory Disease	0	Not present
		1	Present (Asthma, chronic bronchitis, emphysema, removal of one lung (pneumonectomy))

[illegible]

Figure IV.7 continued

Table of Values

Total 1 or Total 2	Value	Total 1 or Total 2	Value	Total 1 or Total 2	Value
-5.0	0.007	-0.9	0.289	3.0	0.953
-4.9	0.007	-0.8	0.310	3.1	0.957
-4.8	0.008	-0.7	0.332	3.2	0.961
-4.7	0.009	-0.6	0.354	3.3	0.964
-4.6	0.010	-0.5	0.378	3.4	0.968
-4.5	0.011	-0.4	0.401	3.5	0.971
-4.4	0.012	-0.3	0.426	3.6	0.973
-4.3	0.013	-0.2	0.450	3.7	0.976
-4.2	0.015	-0.1	0.475	3.8	0.978
-4.1	0.016	0.0	0.500	3.9	0.980
-4.0	0.018	0.1	0.525	4.0	0.982
-3.9	0.020	0.2	0.550	4.1	0.984
-3.8	0.022	0.3	0.574	4.2	0.985
-3.7	0.024	0.4	0.599	4.3	0.987
-3.6	0.027	0.5	0.622	4.4	0.988
-3.5	0.029	0.6	0.646	4.5	0.989
-3.4	0.032	0.7	0.668	4.6	0.990
-3.3	0.036	0.8	0.690	4.7	0.991
-3.2	0.039	0.9	0.711	4.8	0.992
-3.1	0.043	1.0	0.731	4.9	0.993
-3.0	0.047	1.1	0.750	5.0	0.993
-2.9	0.052	1.2	0.769		
-2.8	0.057	1.3	0.786		
-2.7	0.063	1.4	0.802		
-2.6	0.069	1.5	0.818		
-2.5	0.076	1.6	0.832		
-2.4	0.083	1.7	0.856		
-2.3	0.091	1.8	0.858		
-2.2	0.100	1.9	0.870		
-2.1	0.109	2.0	0.881		
-2.0	0.119	2.1	0.891		
-1.9	0.130	2.2	0.900		
-1.8	0.142	2.3	0.909		
-1.7	0.154	2.4	0.917		
-1.6	0.168	2.5	0.924		
-1.5	0.182	2.6	0.931		
-1.4	0.198	2.7	0.937		
-1.3	0.214	2.8	0.943		
-1.2	0.231	2.9	0.948		
-1.1	0.250				
-1.0	0.269				

classified into the three levels of care thus generate demands for nursing services as approximated by the tri-level time estimates derived from McKnight's Colorado Study [41]. Allocation and assignment of resources to meet these demands is the subject of the models proposed in Chapter III.

The factors identified by the methodology used here seem to be in agreement with the findings of other researchers as reported in Chapter II with those few differences that exist being noted. In general, the common items tend to be of the ADL (Activities of Daily Living) and psycho-social variety, with the various identified medically defined conditions seeming to depend on the sample used. In terms of comparison of the predictive and recognitive powers of the system used here with previous work, only the results of Parker and Boyd [53] might provide a rough benchmark. With the total sample of 623 patients scored on the CPAI and all 60 variables in use, a comparison of their discriminant analysis findings for what we have termed Int. B, Int. A, and SNF patients reveals an overall prediction rate of .73 and recognition rate of .79 with about 18% of misses on these groups coming at more than one step off the main diagonal of the classification matrix. Note, however, that with a total of 6 levels being fitted, it rightly

could be said that their model had more opportunities to miss than ours, which is "locked in" for the middle category. Nevertheless, the 12- and 9-variable problems give a reasonably close result to that reported by Parker and Boyd so as to be encouraging.

It should be emphasized that the patient classification system proposed here can only be as accurate a reflection of patient health status as allowed by the data used in its derivation. Therefore, it is recommended that replication of the study be undertaken, utilizing a similar methodological approach with additional data as they become available.

Our purpose has been to demonstrate a reasonably simple methodology and approach to the derivation of nursing service demand data for use in allocation models. In addition, however, the classification scheme can be used to assist the appropriate agencies in patient level of care placement. That is, the results of application of the instrument can serve to "flag" patients who definitely require certain levels of nursing care. This information, in turn, can be combined with other inputs to arrive at an acceptable decision.

The necessity for additional research in the area of subset selection for multivariate methods emphasized by Fisher, _et al_. [23] is reiterated here because

of its obvious importance in both studies of this nature and a variety of other applications.

Chapter V: Mathematical Programming Solution Methodology: Parametric Mixed-Integer Linear Programming

V.1 Introduction

In Chapter III we noted that the examination of solutions of the Basic Staffing Model with alterations in the parameters provides valuable information for both the facility administrator and the director of nursing. Specifically, we saw that by extending the Basic Staffing Model to the forms of Model I and Model II we could interpret the results obtained by parametric changes in the two models to yield insights into the following four aspects of the administrator's problem:

1. Alterations in the staff mix as a result of changing service levels;
2. Alterations in the staff mix as a result of changing budget limits;
3. Changes in legal staffing requirements;
4. Alterations in the priority-related limits on patient demands.

The Basic Staffing Model,

$$\text{MAX} \quad \sum_{i}\sum_{j}\sum_{k} c_{ijk}x_{ijk} + \sum_{j}\sum_{k} p_{jk}\beta_{jk}$$

$$\text{s.t.} \quad \sum_{i} s_i n_i \le \text{Budget}$$

$$\sum_{j}\sum_{k} x_{ijk} - a_i n_i \le 0, \qquad \forall i \qquad \text{(V.1)}$$

$$\sum_{i} x_{ijk} - \beta_{jk} = 0, \qquad \forall j,k$$

$$L_i \le n_i \le U_i \text{ and integer}, \qquad \forall i$$

$$L_{jk} \le \beta_{jk} \le U_{jk}, \qquad \forall j,k$$

$$x_{ijk} \ge 0, \qquad \forall i,j,k,$$

was therefore modified in such a way that the first and second of the four aspects above were subsumed in Model I, while Model II was designed to relate to the remaining considerations. We restate these two models below for purposes of completeness as (V.2) and (V.3).

$$\text{MAX} \sum_{i}\sum_{j}\sum_{k} c_{ijk}x_{ijk} + \sum_{j}\sum_{k} p_{jk}\beta_{jk}$$

$$\text{s.t.} \sum_{i} s_i n_i \le \text{Budget}$$

(Model I)

$$\sum_{j}\sum_{k} x_{ijk} - a_i n_i \le 0, \qquad \forall i$$

$$\sum_{i} x_{ijk} - \beta_{jk} = 0, \qquad \forall j,k \qquad \text{(V.2)}$$

$$\sum_{j}\sum_{k} \beta_{jk} \le S$$

$$L_i \le n_i \le U_i \text{ and integer}, \qquad \forall i$$

$$L_{jk} \le \beta_{jk} \le U_{jk}, \qquad \forall j,k$$

$$x_{ijk} \ge 0, \qquad \forall i,j,k$$

$$\text{MAX} \quad \sum_{i}\sum_{j}\sum_{k} c_{ijk}x_{ijk} + \sum_{j}\sum_{k} p_{jk}\beta_{jk}$$

$$\text{s.t.} \quad \sum_{i} s_i n_i \leq \text{Budget}$$

$$\sum_{j}\sum_{k} x_{ijk} - a_i n_i \leq 0 , \qquad \forall i$$

(Model II)

$$\sum_{i} x_{ijk} - \beta_{jk} = 0 , \qquad \forall j,k \qquad \text{(V.3)}$$

$$\sum_{j}\sum_{k} \beta_{jk} \leq S$$

$$L_i+\theta(L_i^*-L_i) \leq n_i \leq U_i+\alpha(U_i^*-U_i) \text{ and integer}, \ \forall i$$

$$L_{jk}+\theta(L_{jk}^*-L_{jk}) \leq \beta_{jk} \leq U_{jk}+\alpha(U_{jk}^*-U_{jk}), \qquad \forall j,k$$

$$x_{ijk} \geq 0, \qquad \forall i,j,k$$

$$\alpha, \ \theta \in [0,1]$$

We have previously indicated that solutions to each model with alterations in the service level (S) the budget, and the upper and lower bounds via (α,θ), respectively, could be accomplished by resolving each altered problem from scratch. In this chapter, however, we will propose two algorithms, one for each model form, which will allow for the efficient examination of a number of alternative values of S and the budget in Model I and the pair (α,θ) in Model II. This will be accomplished by incorporation of the concepts of parametric programming into an enumerative mixed-integer

linear programming algorithm.

In general, parametric linear programming deals with the study of the optimal solutions to a family of mathematical programs related by a parametric representation of one or more model data components. Included within the general heading of parametric programming are the class of techniques known as "postoptimality analysis." The main feature of these techniques is that the examination of the objective function as a function of the parameters takes place after some previous problem has been solved to optimality with a specific set of parameters. In this way, a minimum of effort relative to complete problem resolution need be expended in finding the new optima. The algorithms to be proposed in this chapter will gain their efficiency by incorporation of the ideas of postoptimality analysis.

For review purposes, the basic ideas of parametric linear programming will first be discussed, followed by an examination of mixed-integer linear programming (MILP) solution techniques. In conjunction with this latter topic, the literature survey of the parametric MILP problems will be presented. Then a basic new parametric MILP algorithm will be given, and its applicability to Models I and II will be discussed. Finally, algorithms designed to solve the generic class

of problems represented by the structures of the two models will be stated, followed by the presentation of computational results for randomly generated example problems. We shall postpone to Chapter VI an example of the use of both algorithms in connection with the solution of the facility administrator's problem.

V.2 Parametric Linear Programming

The parametric MILP algorithm proposed is based on the MILP solution technique known as branch and bound (cf. Garfinkel and Neuhauser [25]). Branch and bound uses linear programming quite extensively, and since we will incorporate the parametric aspects of the new algorithm in the LP routine, we briefly review some of the major concepts of parametric linear programming. For the purposes of the exposition that follows, note that in general the upper and lower bounds on program variables could be explicitly included as model constraints. In this manner, the parametric representation of Model I and Model II can both be subsumed in the parametrization of the right-hand-sides (requirements vector).

After addition of the appropriate slack and surplus variables we may write the general linear program (LP) as

$$\text{(LP)} \qquad \begin{aligned} &\text{MAX } \underline{cx} \\ &\text{s.t. } A\,\underline{x} = \underline{b} \\ &\underline{x} \geq \underline{0} \end{aligned} \qquad \text{(V.4)}$$

where the constraint matrix A is (m x n), the objective coefficients vector $\underline{c}$ is (1 x n), the vector of variables $\underline{x}$ is (n x 1), and the requirements vector $\underline{b}$ is (m x 1). For any set of basic variables $\underline{x}_B$ we have

$$\underline{x}_B = B^{-1}\underline{b} - B^{-1}N\underline{x}_N \geq \underline{0} \qquad \text{(V.5)}$$

where B^{-1} is the inverse of the basis matrix and is (m x m), N is the matrix of non-basic columns of A and is (m x n-m), and $\underline{x}_N$, the vector of non-basic variables, is (n-m x 1). If B is an optimal basis, then $\underline{x}_N = \underline{0}$.

Suppose now that a change in the vector $\underline{b}$ is under consideration. Such a change may include alterations in the entire vector or a subset of its elements. In general, any alteration in the parameters of (LP) can affect either the optimality of a given solution or its feasibility. The optimality and feasibility criteria are:

$$\text{(optimality)} \quad z_j - c_j = \underline{c}_B B^{-1}\underline{a}_j - c_j \geq 0, \quad j=1,2,\ldots,n \qquad \text{(V.6)}$$

$$\text{(feasibility)} \quad \underline{x}_B = B^{-1}\underline{b} \geq \underline{0} \qquad \text{(V.7)}$$

Obviously a change in $\underline{b}$ has no effect in (V.6), but can possibly affect (V.7). As demonstrated by Hadley [28],

if we write the altered requirements vector, $\underline{b}^*$, as

$$\underline{b}^* = \underline{b} + \theta \underline{r} \qquad \text{(V.8)}$$

where for convenience we choose $\theta \in [0,1]$ and $\underline{r}$ to yield the desired range of consideration in the elements of $\underline{b}$ under study, then (V.7) becomes

$$\underline{x}_B^* = B^{-1}\underline{b}^* = B^{-1}(\underline{b}+\theta\underline{r}) \qquad \text{(V.9)}$$

$$= B^{-1}\underline{b} + \theta B^{-1}\underline{r} \geq \underline{0}$$

Thus, $\underline{x}_B^*$ will be non-negative for certain values of θ. In fact, if $B^{-1}\underline{r}$ is non-negative, then the feasibility condition will be met for any $\theta \in [0,1]$. Writing the vector $B^{-1}\underline{r}$ as $\underline{y}$, suppose that some elements of $\underline{y}$ are negative. Then based on (V.9) we can find that value of θ, called θ_c, such that some $x_{B_i}^* < 0$ for $\theta > \theta_c$, where

$$\theta_c = \min_i \left[\frac{-x_{B_i}}{y_i} \right], \quad y_i < 0 . \qquad \text{(V.10)}$$

Then for $\theta > \theta_c$, the optimality condition is satisfied, but the feasibility condition is violated. For any specific value of $\theta > \theta_c$, the optimal solution can be found (if one exists) using the dual simplex algorithm, starting from the optimal basis obtained earlier. Thus, if only certain values of θ, say $\theta_1, \theta_2, \ldots, \theta_k$ are of interest, the feasibility condition (V.9) can be checked for these values. If the feasibility condition is violated, we use the previous basis to reoptimize as

before.

An interesting property of the type of post-optimality analysis under consideration here is given without proof in Theorem 1, first postulated by Dantzig [21].

<u>Theorem 1</u>: For the problem (LP), the objective function $Z = \text{MAX } \underline{c}\ \underline{x}$ is a piecewise-linear, concave function of the requirements vector $\underline{b}$.

We observe that such a strong result cannot be obtained in the case of the MILP (for further analysis of MILP continuity properties, see Radke [55]).

A further consideration concerns the practicality of simultaneous parameterization of subsets of the requirements vector in terms of two or more parameters. For example, consider the problem posed below:

$$\begin{aligned} &\text{MAX } \underline{c}\ \underline{x} \\ \text{s.t. } &A_1\underline{x}_1 = \underline{b}_1 + \theta\ \underline{r}_1 \\ &A_2\underline{x}_2 = \underline{b}_2 + \lambda\ \underline{r}_2 \qquad \text{(V.11)} \\ &\underline{x} \geq \underline{0} \\ &\theta, \lambda \in [0,1] \end{aligned}$$

The feasibility condition (V.9) requires that

$$B^{-1} \begin{bmatrix} \underline{b}_1 \\ \cdots \\ \underline{b}_2 \end{bmatrix} + B^{-1} \begin{bmatrix} \theta \underline{r}_1 \\ \cdots \\ \lambda \underline{r}_2 \end{bmatrix} \geq \underline{0} \ . \tag{V.12}$$

Partitioning B^{-1} to be compatible with the row partition of the parametric alterations vector as

$$B^{-1} = [D_1 \vdots D_2] \ , \tag{V.13}$$

we may write (V.12) as

$$B^{-1}\underline{b} + \theta D_1 \underline{r}_1 + \lambda D_2 \underline{r}_2 \geq \underline{0} \ . \tag{V.14}$$

The obvious difficulty is that each one of the above equations in the two unknowns θ and λ makes the derivation of a condition analogous to (V.10) difficult. This result has implications for our study of Model II, where simultaneous parameterization of the upper and lower bounds is proposed. The combinatorial nature of such parameterizations may be resolved by fixing one parameter a priori thus reducing (V.14) to the single parameter case as in (V.9).

In this section we have briefly outlined some important results of parametric linear programming. These results, with modifications, will form the basis for the parametric branch and bound algorithms to follow. Note that due to the nature of Models I and II we have only considered alterations in the requirements vector $\underline{b}$.

There exist equally well-developed methodologies for dealing with the parameterization of the objective function coefficients vector $\underline{c}$ and the elements of the constraint matrix A. For a detailed explanation of these results the reader may consult Hadley [28].

V.3 Parametric Integer and Mixed Integer Linear Programming

When all or a subset of the variables of a linear program are constrained to be integers, there is a considerable increase in difficulty in solving the problem. It is well known that this is essentially due to the loss of the properties of the LP feasible region upon which the simplex algorithm is based. That is, there is no longer any guarantee that the optimal solution will lie at an extreme point of the feasible region. Obviously, therefore, the postoptimality analysis results applicable in the LP case cannot be extended to the case of the integer linear program (ILP) or MILP. Under some very special circumstances, however, the application of the simplex algorithm and subsequent LP postoptimality analysis will be appropriate. If the sufficient condition of total unimodularity (cf. Garfinkel and Neuhauser [25], of the ILP or MILP constraint matrix A is met, LP results may be applied to yield feasible solutions. Then, too,

it may be the case that A possesses some other special structure which allows the use of LP.

Because of the general inappropriateness of the simplex method for use in the MILP, various other solution techniques have been developed (cf. [25], [60]). These techniques essentially fall into three major classes:

1. Cutting planes;
2. Benders partitioning;
3. Branch and bound.

Based on the theory of each of these methodologies, several authors ([26], [1], [6], [7], [59], [49]) have proposed extensions to deal with the questions of postoptimality analysis with varying degrees of success. We shall therefore briefly review each solution technique, indicating also the subsequent efforts in the realm of postoptimality analysis. Note that the efforts to be discussed in connection with cutting planes and Benders partitioning are concerned with closing the "duality gap" and the concept of "sensitivity analysis," both to be defined below. While not directly applicable to our work, the review of such efforts serves not only to give historical perspective, but also to indicate how postoptimality analysis might be carried out for small changes in the requirements vector. We therefore consider the

techniques of sensitivity analysis as a subclass of the techniques of postoptimality analysis.

V.3.1 Cutting Planes

The cutting plane approach, primarily due to Gomory, is based on the philosophy of adding additional constraints, called "cuts," on the feasible region of the LP relaxation of the MILP. The cut constraint formation is based on information contained in any row of the simplex tableau (called the "source row") and such constraints are appended to the tableau. In general, the additional constraint will cause primal infeasibility, necessitating the application of the dual simplex algorithm. It is guaranteed that no mixed-integer feasible solution will be excluded by application of the cuts. For further details on the specific methodology, the reader is referred to Garfinkel and Nemhauser [25] or Salkin [60]. The mathematics of cutting plane algorithms are quite elegant, leading one to believe the methods might be computationally attractive. This assumption has not been borne out in practice, however, where it is often the case that the finite number of steps needed for algorithmic termination are, in fact, unachievable in a practical sense.

Because the cutting plane method is based on the solution of the LP relaxation of the MILP, with the

constraint set expanded by the addition of cuts, it is natural to conjecture that the resulting optimal LP might be useful in performing postoptimality analysis. One such attempt was made by Gomory and Baumol [26] for the ILP, but the analysis was somewhat different from what we have to this point called postoptimality analysis. The Gomory and Baumol approach is based on examination of a "sensitivity analysis" of the final simplex tableau at termination of the cutting plane algorithm; that is, how sensitive is the optimal objective function value to marginal changes in the requirements vector. Such information may be obtained from an examination of the dual variables, available as the non-basic slack variables of the primal LP simplex tableau or from solution of the dual linear program. A well-known result with respect to the primal and dual LPs is given without proof in Theorem 2.

Theorem 2: If the primal and dual linear programs possess feasible solutions $\underline{x}$ and $\underline{y}$ respectively, then $\underline{x}$ and $\underline{y}$ are also optimal to their respective problems, and further,

$$\underline{cx} = \underline{yb} \tag{V.15}$$

where $\underline{yb}$ is the dual objective function.

In the case of the MILP, Theorem 2 has no direct interpretation since the dual for the MILP is not defined.

Nevertheless, in the solution of the LP relaxation of the MILP,(V.15) is true. In general, however,

$$\underline{cx}_{MILP} \leq \underline{cx}_{LP} \tag{V.16}$$

Thus the difference between the objective function values is called the "duality gap." If such a gap exists, the results of LP sensitivity analysis cannot be applied. The Gomory and Baumol approach is to use the dual variables associated with the cut constraints to recompute the dual variables associated with the original constraints.

By "imputing back" the effect ofthe dual variables obtained using the cuts to the duals associated with the original constraints, the relation (V.15) was shown to be true for active (or tight) original constraints only. Although in the usual LP problem inactive constraints have a zero dual variable associated with them, this property may or may not be true as a result of recomputation. Further, the recomputed duals are non-unique, depending upon the order of cut generation. Thus, the Gomory and Baumol approach does not completely solve the sensitivity analysis question.

Some years later, Alcaly and Klevorick [1], recognizing the shortcomings of the Gomory and Baumol proposal, asserted a system of "generalized dual prices" which remedied the problems with inactive constraints. Nevertheless, the duality gap was shown to exist even

under this new methodology, leaving ILP sensitivity analysis an open question.

V.3.2 Benders Partitioning

The Benders partitioning method of MILP solution relies on the use of information obtained by iteratively solving two problems partitioned from the original MILP. Following the exposition given in Garfinkel and Nemhauser [25], we write the initial MILP as

$$\begin{aligned} \text{MAX} \quad & \underline{c}_1\underline{x} + \underline{c}_2\underline{y} \\ \text{s.t.} \quad & \underline{A}_1\underline{x} + A\,\underline{y} \leq \underline{b} \\ & \underline{x} \geq \underline{0} \text{ and integer} \\ & \underline{y} \geq \underline{0} \end{aligned} \tag{V.17}$$

The Benders LP problem is

$$\begin{aligned} \text{MAX} \quad & \underline{c}_1\underline{x} + \min_{\underline{u}} \; u(\underline{b} - A_1\underline{x}) \\ & \underline{x} \geq \underline{0} \\ \text{s.t.} \quad & \underline{u}\, A_2 \geq \underline{c}_2 \\ & \underline{u} \geq \underline{0}\,, \end{aligned} \tag{V.18}$$

and is obtained by taking the dual of (V.17) with $\underline{x}$ fixed at some value, thus reducing the MILP to an LP.

The second Benders problem, an ILP, is given as

$$\begin{array}{lll} \text{MAX} & u_o & \\ \text{s.t.} & u_o \le \underline{c}_1\, \underline{x} + \underline{u}^t(\underline{b}-A_1\underline{x}) & ,\forall\, \underline{u}^t \in T \\ & \underline{0} \le \underline{y}^q(\underline{b}-A_1\underline{x}) \ , & \forall\, \underline{y}^q \in Q \\ & \underline{x} \ge \underline{0} \text{ and integer} & \end{array} \qquad (V.19)$$

where $T = (\underline{u}^t \mid \underline{u}^t$ is an extreme point of the constraint set of (V.18)) and $Q = (\underline{y}^q \mid \underline{u}^t + \theta\underline{y}^q,\ \theta \ge 0$, is an extreme ray for some $\underline{u}^t \in T)$. Benders' algorithm calls for the iterative application of (V.19) to determine an $\underline{x}$ to be used in (V.18), from which either an optimal solution is obtained or the sets T and/or Q are altered. Problem (V.19) is again solved, and so on. The algorithm can be shown to be finite.

Based on the concepts of the partitioning method, Balas [7,8] proposed a constructive approach for solving the duality gap problem, thereby allowing resolution of the MILP sensitivity analysis question. As the initial step, a dual problem to (V.17) is defined as follows:

$$\begin{array}{ll} \underset{\underline{x}}{\text{MAX}}\ \underset{\underline{u}}{\text{MIN}} & \underline{c}_1\underline{x} + \underline{u}(\underline{b}-A_1\underline{x}) \\ \text{s.t.} & \underline{u}\, A_2 - \underline{v} = \underline{c}_2 \\ & \underline{u} \ge \underline{0} \\ & \underline{x} \ge 0 \text{ and integer} \\ & v_j \text{ unconstrained, } j \in N_1 \\ & v_j \ge 0,\ j \in N_2\ , \end{array} \qquad (V.20)$$

where N_1 is the index set of $\underline{x}$ and N_2 the index set of $\underline{y}$. The solution of the MILP (V.20) yields $\underline{v}$, from which a vector of "subsides," $\underline{s}$ may be derived. These subsidies, in turn, lead to the construction of the following special LP, formed from the relaxation of (V.17):

$$\begin{aligned} &\text{MAX } (\underline{c}_1+\underline{s})\underline{x} + \underline{c}_2\underline{y} \\ &\text{s.t. } A_1\underline{x} + A_2\underline{y} \leq \underline{b} \\ &\underline{x},\ \underline{y} \geq \underline{0}\ , \end{aligned} \tag{V.21}$$

Balas demonstrates that

$$(\underline{c}_1+\underline{s})\underline{x} + \underline{c}_2\underline{y} = \underline{ub}\ , \tag{V.22}$$

and thus the duality gap is closed. This is because an optimal solution to (V.17) will also be optimal to (V.21), and the dual variables of (V.21) are the proper values to use in a sensitivity analysis of the original MILP. The connection with Benders partitioning method is clear when (V.20) is examined. Assuming we fix $\underline{x}$ to be "admissible," (V.20) is the dual LP analogous to (V.18). The appropriate analog for (V.19) may also be derived.

In spite of the fact that the MILP sensitivity analysis question appears to be answered, this method nevertheless has its drawbacks. Of major concern is that solution of (V.21) will not, in general, yield an MILP feasible solution. The reorientation of the objective

function simply assures that the MILP solution may be an alternate optimum. Any change in $\underline{b}$ would be reflected in the solution for $\underline{v}$, thereby changing $\underline{s}$. Hence, even if we could guarantee finding the proper solution, (V.21) may not be helpful in solving the postoptimality analysis problem. Attempts in connection with our study to find an efficient scheme for updating $\underline{v}$ could not be found, any such alteration requiring a complete resolution of (V.20), which is tantamount to resolving the MILP from the outset with the parametric change incorporated.

V.3.3 Branch and Bound

The method of Branch and Bound is probably the most widely applied MILP solution technique. Recognizing the essentially combinatorial nature of the MILP, this method seeks to identify those solutions which are dominated by others and to remove them from further consideration. Dominance is inferred from upper and lower bounds on the objective function obtained via the solution of LP relaxations.

If solution of the relaxation of the original MILP does not yield an MILP feasible solution (if it does, we are done), an integer-restricted variable in the optimal solution which did not terminate integer is chosen. Calling this variable x_i^*, two new LPs are formed

with constraint sets

$$R_1 = \left((\underline{x},\underline{y}) \middle| A_1\underline{x} + A_2\underline{y} \leq \underline{b};\ \underline{x},\underline{y} \geq \underline{0},\ x_i \geq [x_i^*] + 1\right)$$

$$R_2 = \left((\underline{x},\underline{y}) \middle| A_1\underline{x} + A_2\underline{y} \leq \underline{b};\ \underline{x},\underline{y} \geq \underline{0},\ x_i \leq [x^*]\right)$$

This partitioning insures that no feasible solutions will be excluded. The LP with either R_1 or R_2 is chosen for solution, and the method proceeds in this fashion. An upper bound $\overline{Z}_i$ at problem i is obtained from the feasible LP objective value. A lower bound, $\underline{Z}_i$, can be found from a MILP feasible solution. The maximal such lower bound is called $\underline{Z}_o$. For any problem with constraint set R_i, we may curtail further consideration of it and its successor problems if

$$\overline{Z}_i = \underline{Z}_i, \text{ or} \qquad \text{(V.23)}$$

$$\overline{Z}_i \leq \underline{Z}_o \qquad \text{(V.24)}$$

The condition (V.23) implies that no better MILP feasible solution can be found, while (V.24) yields the conclusion that the best known MILP feasible solution is better (has higher objective function value) than any succeeding LP relaxation. Both conditions result from the fact that the successor to a given problem is more constrained than the problem itself.

The choice of variables upon which to partition, the order of problem consideration, and the improvement of bounds are all subjects of algorithmic improvement and

will not be considered here. Again, the reader is referred to the texts mentioned earlier.

The parametric methodologies we have discussed in connection with the cutting plane and Benders partitioning algorithms have been concerned with the question of obtaining sensitivity analysis results for the MILP by resorting to some modified LP form. Utilizing the branch and bound algorithm, however, two authors, Roodman [59] and Nauss [49], have been able to provide insights into the more general question of MILP postoptimality analysis.

The method of Roodman [59] is essentially a modification of the branch and bound algorithm designed to extract sufficient information such that

1. The critical values for parametric representation of the requirements vector and/or the objective function coefficients may be determined; and
2. Efficient problem resolution after a specific parametric alteration in the requirements vector and/or objective function coefficients is possible.

A crucial factor in the application of the method is the initial statement of the MILP in the form of the most restrictive problem to be considered. That is, for the

problem (V.17), $\underline{c}^+$ and $\underline{b}^+$ are taken to be the minimum possible values. In addition, the MILP must be transformed so that the integer-restricted variables are binary, and the continuous variables are defined on the interval [0,1].

Initially, the restricted MILP is solved to optimality using branch and bound. The reason for fathoming (deleting from further consideration a problem and its successors) each node is noted as being due to one of the following three conditions:

1. Bounds;
2. Integrality;
3. Infeasibility.

For conditions (1) and (2), critical values of the parameters involved in the objective function and requirements vector such that the fathoming condition is just violated are determined. For condition (3), a critical value for the requirements vector such that the problem would become feasible is found. All such information for every fathomed node must be stored. The minimum, taken over all nodes, of the first two of these values gives the critical values for the objective function and requirements vector parameters such that below these values the current optimal solution remains optimal. For a proposed parametric change, the appropriate

critical value is considered for each node (depending on the fathoming condition) to determine if it is violated. If so, these problems, with the change incorporated, must be resolved, and the branch and bound algorithm continued from these nodes.

In retrospect, Roodman's method captures the essence of the MILP postoptimality analysis problem. Unfortunately, it has several limitations, among which are the following:

1. The requirement for initial transformation of the MILP;
2. Potentially large amounts of information to be stored (e.g., fathoming conditions, critical values, etc., for every node);
3. LPs to be resolved must be started from scratch (if, for example, the optimal bases are retained, this greatly increases storage requirements);
4. No use is made of LP parametric programming techniques.

When taken together, it appears that these factors detract from the computational efficiency of the algorithm.

In a recent study, Nauss [49] considered the parametric integer linear programming problem (PILP) for

the following two cases:

1. Parameterization over a finite number of points;
2. Varying one parameter continuously over a specified range.

The general (PILP) is proposed as

$$\begin{array}{lll} & \text{MIN} & (\underline{c}+\underline{f}_k)\underline{x} \\ (\text{PILP}) & \text{s.t.} & (A+D_k)\underline{x} \geq \underline{b} + \underline{r}_k \\ & & \underline{x} \geq \underline{0} \text{ and integer}; \quad k=1,2,\ldots,k^*. \end{array} \tag{V.25}$$

The solution of PILP is based on the branch and bound algorithm, modified by "problem dependent techniques" and "solution priorities" to be discussed below. The ultimate application of Nauss' analysis is to each of three specific PILP problems: the generalized assignment problem, the 0-1 knapsack problem, and the capacitated facility location problem.

Initially, a rudimentary and general algorithm is given for the solution of (V.25). Defining

P_{k,R_i} = PILP indexed on k with R_i implying a given restriction on the problem at node i (R_o is the original constraint set).

The method proceeds as in the branch and bound algorithm, but all k* problems are, in general, considered at every node. If a fathoming condition is encountered for the k^{th} problem, consideration of this problem may be

deleted at successor nodes. Several open questions exist in connection with the PILP algorithm, among them

1. The form of the relaxation of P_{k,R_i} to employ,
2. Any initial setup of PILP to take maximum advantage of problem structure,
3. The branching rule to invoke, and how to apply it to the remaining problems at a node.

No specific procedures are advanced to answer these questions.

Nauss proposes several problem dependent techniques to improve algorithmic efficiency for specific problem structures. The first of these is "problem reduction," attained by pegging as many variables as possible to their optimal solution values and eliminating constraints found to be dominated by others. This result is most powerful when the reduction is applicable to all problems of the PILP. "Feasibility recovery" involves the use of an existing optimal solution to the k^{th} problem to generate a feasible solution (and upper bound) for the $k+1^{st}$ problem. Third, "bounding problem reoptimization" is indicated in algorithms using LP as the relaxation. The author proposes that LP parametric programming techniques such as those discussed in Section V.2 might be a way to solve the relaxations of all problems of the PILP. Finally, the author suggests "wide range bounding" as a technique involving the use of the

Lagrangean dual problems to generate upper bounds for the problems of PILP.

Solution priorities deal with the distribution of effort in solving the PILP; that is, how we should go about solving the k* problems. The "purely serial" approach calls for solution of one problem to optimality, then gleaning as much information as possible from it to assist in solving the next problem, and so on. Information such as the separation of variables, upper and lower bounds, fathoming conditions, etc., can be helpful, assuming the course of enumeration for the next problem is similar to that of its predecessor (note that Roodman's algorithm is an example of the purely serial approach). The "lexicographically serial" approach is based on the continued solution of problem 1 to those nodes where a fathoming condition is encountered for it, then switching to problem 2, etc., until all k* problems are considered. The final solution priority, the "parallel" approach, has all k* problems, each with the same R_i, under consideration at every node. When a fathoming condition for problem k is met at a node, the problem is not considered at successors to that node. Ultimately, when all problems remaining at a node have been fathomed, the node itself is fathomed. Nauss states that, "the parallel approach...relies on the assumption

that solving a series of closely related problems at a given node can be accomplished relatively efficiently." [49, p. 38] He recommends the first two priorities over the parallel approach.

An interesting result due to Noltemeier [51] and cited by Nauss is given as Theorem 3:

<u>Theorem 3</u>: (Noltemeier) Given that $\underline{c}$, A, and $\underline{b}$ are integer quantities, then the PILP parameterized in the requirements vector only may be transformed into an equivalent PILP of the form

$$\begin{aligned} \text{MIN} \quad & \underline{cx} \\ \text{s.t.} \quad & A\underline{x} \geq \underline{b} + \underline{t}_k \qquad \text{(V.26)} \\ & \underline{x} \geq \underline{0} \text{ and integer} \end{aligned}$$

where $\underline{t}_k$ is an integer vector conformable with $\underline{b}$ for k=1,2,...,K*.

As a result of this theorem, continuous parameterization of the requirements vector may be reduced to the discrete case for the PILP. This result is not true, in general, in the parametric mixed-integer linear programming case.

The work of Nauss is a major effort in parametric programming. Little attention is given, however, to the mixed-integer case. In addition, several open questions pertaining to algorithmic implementation remain. These

questions were answered for the three problem forms mentioned earlier, but not for general PILP. As a final note, Nauss also recognized the difficult combinatorial problem implied by simultaneous parameterization of program data with different parameters, but did not recommend procedures to alleviate the difficulties.

V.4 A Proposed MILP Parametric Programming Algorithm

We now propose an enumerative algorithm for solving the following parametric mixed-integer linear program (PMILP).

$$\begin{aligned} \text{MAX} \quad & \underline{c}_1\underline{x} + \underline{c}_2\underline{y} \\ \text{s.t.} \quad & A_1\underline{x} + A_2\underline{y} = \underline{b}_k \qquad \text{(V.27)} \\ \text{(PMILP)} \quad & \underline{x} \geq \underline{0} \text{ and integer} \\ & \underline{y} \geq \underline{0} \\ & k=1,2,\ldots,K^* \end{aligned}$$

We call the individual problems of (V.27) (P_k), and the LP relaxations of these problems (RP_k). A parallel solution approach will be taken. Further, we assume that as the index k increases, the problems of the PMILP become more restrictive. Note that the form of (V.27) implies that the continuous parameterization of $\underline{b}$ has been reduced to the discrete case. In the absence of the result of Theorem 3, the discretization is certainly arbitrary,

[illegible]

[illegible]

$$[illegible]$$

[illegible]

and as such may be chosen in accordance with the problem setting.

In our modified branch and bound algorithm, we wish to consider $(RP_k)^j$ for all k at any node j. Let (x_j^k, y_j^k) be a solution to the k^{th} relaxed problem at node j. Then if we write the constraint set of $(RP_k)^j$ (including additional restrictions due to branching) as T_j^k and that of $(P_k)^j$ as S_j^k, it is immediate that

$$(\underline{x}_j^k, \underline{y}_j^k) \in T_j^k \Rightarrow (\underline{x}_j^k, \underline{y}_j^k) \in S_j^k \ , \qquad \text{(V.28)}$$

since, due to the nature of the LP relaxation

$$T_j^k \supseteq S_j^k \ . \qquad \text{(V.29)}$$

We can write

$$Z(\underline{x}, \underline{y}) = \underline{c}_1\underline{x} + \underline{c}_2\underline{y} \ , \qquad \text{(V.30)}$$

from which we can infer the following relationships for our problem:

$$Z_j^{*k} = \begin{cases} Z(\underline{x}_j^{*k}, \underline{y}_j^{*k}) \text{ if } (\underline{x}_j^{*k}, \underline{y}_j^{*k}) \text{ is optimal to } (P_k)^j \\ -\infty \text{ if } S_j^k = \emptyset \\ \infty \text{ if } (P_k)^j \text{ is unbounded} \end{cases}$$

Due to (V.28) and (V.29) we can obtain an upper bound, $\overline{Z}_j^k$, on Z_j^{*k} by solving $(RP_k)^j$ to yield

$$\bar{z}_j^k = \begin{cases} z_j^{ok} = z(\underline{x}_j^{ok}, \underline{y}_j^{ok}) \text{ if } (\underline{x}_j^{ok}, \underline{y}_j^{ok}) \text{ is optimal to } (RP_k)^j \\ -\infty \text{ if } T_j^k = \emptyset \\ \infty \text{ if } (RP_k)^j \text{ is unbounded} \end{cases}$$

With these concepts in mind, we now state a basic branch and bound algorithm for PMILP.

<u>Basic Algorithm</u>

Step 1: (Initialization) Start at vertex $j=1$ with $\bar{z}_o^k = \infty$ and $\underline{z}_o^k = -\infty$ $\forall$ k. Set $S_1^k = S^k$ $\forall$ k, where S^k is the initial constraint set for problem k, p=1, and go to step 4.

Step 2: (Branching) If there do not exist any live vertices, go to step 7; otherwise select a live vertex j and go to step 4.

Step 3: (Separation) Choose the lowest indexed problem k for which $(\underline{x}_j^{ok}, \underline{y}_j^{ok}) \notin S_j^k$ and partition S_j^k into two new sets with corresponding live nodes by taking

$S_{j+1}^k = S_j^k \cap (\underline{x}^k \mid x_{B_i}^k \leq [x_{B_i}^{ok}])$ and

$S_{j+2}^k = S_j^k \cap (\underline{x}^k \mid x_{B_i}^k \geq [x_{B_i}^{ok}] + 1)$,

where $x_{B_i}^{ok}$ is some integer-constrained variable such that $x_{B_i}^{ok} \neq [x_{B_i}^{ok}]$. Set $j=j+2$ after partitioning. Choose the same partition $\forall$ k and go to step 2.

Step 4: (LP solution) Solve the set of LPs $(RP_k)^j$ $\forall$ k under consideration, say $k \geq p$, at vertex j. If $\nexists$ a feasible solution to $(RP_p)^j$ set $\bar{z}_j^k = -\infty \; \forall k \geq p$, fathom the node, and go to step 2. In solving the problems $(RP_k)^j$ for any subsequent $\hat{k} > p$, should an infeasible solution be encountered set $\bar{z}_j^k = -\infty \; \forall k \geq \hat{k}$. Otherwise, set $\bar{z}_j^k = z_j^{ok}$; update p as appropriate.

Step 5: (Integrality test) If $(\underline{x}_j^{ok}, \underline{y}_j^{ok})$ is feasible to $(P_k)^j$ $\forall$ k, set $\underline{z}_j^k = z_j^{ok}$, and $\underline{z}_o^k = \text{MAX}\ (\underline{z}_o^k, \underline{z}_j^k)$, fathom the node and go to step 2. If $(\underline{x}_j^{ok}, \underline{y}_k^{ok})$ are feasible to $(P_k)^j$ for a subset of $1 \leq k \leq k^*$, set bounds as above for these values of k, delete the corresponding problems from further consideration at successors of node j.

Step 6: (Bounds) If $\bar{z}_j^k \leq \underline{z}_o^k \; \forall k$, fathom the node j and go to step 2. If this condition is met for a subset of $1 \leq k \leq k^*$, the corresponding problems may be deleted from further consideration at successors to node j. Go to step 3.

Step 7: (Completion) Terminate. For those values of k such that $\underline{z}_o^k = -\infty$ there is no feasible solution. Otherwise, for those k such that $\underline{z}_o^k > -\infty$, the solution which yielded the terminal value of $\underline{z}_o^k$ is optimal for the respective problem k.

The following comments concerning the algorithm are useful:

1. It is convenient to choose the first available live node in backtracking in step 2.
2. Identical separations are invoked $\forall$ k in step 3.
3. The solution of the LPs at every node in step 4 should be accomplished in as efficient manner as possible.
4. The increasingly restricted nature of the problems implied by increasing k does not allow for "feasibility recovery," but rapid detection of infeasible problems can enhance a "bounding problem reoptimization" approach. Exploration of this concept is the essence of Section V.5.

Finiteness of the algorithm is assured if it is assumed that each element of $\underline{x}$ is bounded from above. This implies some maximum number of possible solutions

for each k, and hence the total number of solutions for all the problems put together is finite.

V.5 The Upper Bounded Dual Simplex Algorithm

Turning to a discussion of the efficient solution of the LP relaxations in the Basic Algorithm, several factors weigh on the decision concerning the use of specific techniques. These include:

1. The desire to eliminate potentially large numbers of trivial upper and lower bound constraints so as to reduce storage requirements as much as possible.
2. The need for a method of regaining primal feasibility in a simplex tableau, having become infeasible due to a parametric change in the requirements vector and/or the addition of constraints due to separation and branching.
3. The expectation that the several problems under consideration at a node may be efficiently solved using LP parametric analysis.

The concepts in item (1) may best be incorporated by use of the well known primal upper-bounded simplex algorithm (cf. Hadley [28], Zionts [86], Garfinkel and Nemhauser [25]). Conceptually, this method adds a bookkeeping

device to the simplex algorithm to deal with the upper bounds. Lower bounds may be handled by simple transformation of variables. The requirement of item (2) is best met by use of the dual simplex method, since the optimality conditions will remain satisfied. Both of these techniques are combined in the Upper Bounded Dual Simplex (UBDS) algorithm due to Wagner [76]. After examining this method, we shall derive the rules required for parametric analysis.

Because the UBDS algorithm is designed for upper-bounded variables only, lower bounds must be eliminated by a transformation of variables. Thus, after adding slacks and surplus variables as appropriate, we obtain the LP

$$\begin{aligned} &\text{MAX } \underline{cx}' \\ &\text{s.t. } A\underline{x}' = \underline{b}' \\ &\underline{L} \leq \underline{x}' \leq \underline{\mu}' \, . \end{aligned} \tag{V.31}$$

By defining

$$\begin{aligned} \underline{x}' &= \underline{x} + \underline{L} \\ \underline{b} &= \underline{b}' - A\underline{L} \\ \underline{\mu} &= \underline{\mu}' - \underline{L} \end{aligned} \tag{V.32}$$

and making the appropriate transformations, we obtain the LP given below as (V.33)

[illegible]

$$\begin{aligned} &\text{MAX } \underline{c}\underline{x} \\ &\text{s.t. } A\underline{x} = \underline{b} \\ &\underline{0} \leq \underline{x} \leq \underline{\mu} \end{aligned} \tag{V.33}$$

Assuming all x_i have an upper bound μ_i, we define a "complementary variable x_i'' such that

$$\underline{x} + \underline{x}'' = \underline{\mu} . \tag{V.34}$$

Proceeding with the derivation as shown by Wagner, the system of constraints of (V.33) is written as

$$\sum_{i=1}^{n} x_i \underline{p}_i = \underline{b} \tag{V.35}$$

where $\underline{P}_i$ are the columns of A. The essential substitution to be made is as follows: If variable x_i has its corresponding $c_i > 0$, then substitute the complementary variable into (V.35) and into the objective function,

$$z = \sum_{i=1}^{n} c_i x_i \tag{V.36}$$

The modified forms of (V.35) and (V.36) with, say, d variables having $c_i > 0$ are

$$z = \sum_{i=1}^{d} (-c_i) x_i'' + \sum_{i=d+1}^{n} c_i x_i + \sum_{i=1}^{d} c_i \mu_i \tag{V.37}$$

and

$$\sum_{i=1}^{d} (-\underline{p}_i) x_i'' + \sum_{i=d+1}^{n} x_i \underline{p}_i = \underline{b} - \sum_{i=1}^{d} \underline{p}_i \mu_i \tag{V.38}$$

In establishing the initial tableau, the coefficients of the x_i and x_i'' in (V.38) form the columns. An initial

[illegible]

basis is formed from the slack and/or artificial variables. Obtaining an initial dual feasible solution is easy, since all c_i are non-positive. Thus if no artificials are present, $\underline{c}_B = \underline{0}$ and

$$z_j - c_j = \underline{c}_B B^{-1} \underline{a}_j - c_j = -c_j \geq 0 \qquad \text{(V.39)}$$

When artificials are used, the form of $\underline{c}_B$ is $(0,\ldots,0,-M,\ldots,-M)$, where M is a large positive constant. It is not, therefore true, in general, that $z_j \geq 0$. In this case, for a non-basic j such that $z_j < 0$, the corresponding column (including the entry for $z_j - c_j$) is transformed by multiplying by (-1), thus assuring $z_j - c_j \geq 0$. This is equivalent to the substitution of the complementary variable. The UBDS algorithm as stated by Wagner is given below:

Step 1: (Exit rule) Choose the variable to leave the basis from

$$x_{B_r} = \min_{i=1,2,\ldots,m} (x_{B_i}), \; x_{B_i} < 0.$$

If all x_{B_i} are non-negative, stop. An optimal solution has been obtained. Otherwise, go to step 2.

Step 2: (Enter rule) Choose the variable to enter the basis from

$$x_k = \min_{j=1,2,\ldots,n} [(z_j - c_j)/-x_{rj}], \; x_{rj} < 0$$

[illegible]

[illegible]

[illegible]

If $\nexists\, x_{rj} < 0$, stop. The dual is unbounded which implies the primal problem has no feasible solution. Otherwise, go to step 3.

Step 3: (Initial new tableau) Perform a transformation on the current tableau using the usual pivoting rules of the simplex method. Go to step 4.

Step 4: (Final new tableau) For any basic variable, say x_{B_t}, which violates its upper bound:

1. Change the signs of all elements in the tableau row corresponding to x_{B_t} except the coefficient 1 in the column denoting x_{B_t}.
2. Replace x_{B_t} by its complementary variable.
3. Label x_{B_t} as x''_{B_t} (or vice-versa as the case may be).
4. Change the sign of the c_t corresponding to x_{B_t}. Go to step 1.

Upon algorithmic termination, the solution may be decoded by first noting that variables will belong to one of the following four mutually exclusive and collectively exhaustive classes:

1. Basic $x_i \Rightarrow x_i = x_i$

2. Non-basic $x_i \Rightarrow x_i = 0$

3. Basic $x''_i \Rightarrow x_i = \mu_i - x''_i$

4. Non-basic $x''_i \Rightarrow x_i = \mu_i$

Thus, aside from some rather easy transformations and decoding rules, the algorithm is much like the ordinary dual simplex algorithm.

With the UBDS algorithm providing an efficient method for solving the general LP relaxation encountered at any node, we turn to the consideration of solving the closely related relaxed problems. This implies the need for a postoptimality analysis technique based on an optimal UBDS tableau. As demonstrated earlier in Section V.2, the basis for the total problem is necessary for sensitivity and/or parametric analysis. For this reason we initially examine the structure of upper bounded problems in a manner suggested by Hadley [28].

Beginning with the LP form (V.33), the so-called "expanded system" of constraints with upper bounds and complementary variables explicitly included is

$$\begin{bmatrix} A & O \\ I_n & I_n \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{x}'' \end{bmatrix} = \begin{bmatrix} \underline{b} \\ \underline{\mu} \end{bmatrix} \quad \underline{x},\ \underline{x}'' \geq \underline{0}\ , \qquad \text{(V.40)}$$

where I_n is the identity matrix of order n and A is

(m+n) x (m+n).

Hadley demonstrates the following three results.

Result 1: A basis for the system (V.40) does not contain more than m variables which are not at their upper bounds.

Proof: Assume there exist $k > m$ such variables. Then there also exist k corresponding complementary variables in the basis. Also, there will be n-k additional vectors in the basis. Therefore, the basis will consist of $n-k+k+k=n+k > n+m$ variables, which is impossible. QED

Result 2: The basis matrix for (V.40) may be constructed in partitioned form as

$$B_* = \begin{bmatrix} B & O & O & R \\ I_m & I_m & O & O \\ O & O & I_{n-m-k} & O \\ O & O & O & I_k \end{bmatrix} \qquad (V.41)$$

where B_* is $(m+n)x(m+n)$, B is (mxm) and is the basis for (V.33) consisting of columns corresponding to the r variables not at their upper bounds. If $r < m$, the remaining m-r columns are filled with the columns corresponding to variables at their upper

bounds. Columns m+1 to 2m contain the complementary vectors for the variables in columns 1 through m. The final k columns consist of the columns of remaining variables at their upper bounds, if any, with n-m-k being the remaining columns, if any.

Result 3: The inverse of the basis matrix B_* may be written in partitioned form as

$$B_*^{-1} = \begin{bmatrix} B^{-1} & 0 & 0 & -B^{-1}R \\ -B^{-1} & I_m & 0 & B^{-1}R \\ 0 & 0 & I_{n-m-k} & 0 \\ 0 & 0 & 0 & I_k \end{bmatrix} \qquad (V.42)$$

Results 1-3 will be useful when attempting to determine specific rules for postoptimality analysis. Further, we may delineate two instances where B_*^{-1} will take on a special form. These are

Case I: k=0

$$B_*^{-1} = \begin{bmatrix} B^{-1} & 0 & 0 \\ -B^{-1} & I_m & 0 \\ 0 & 0 & I_{m-n} \end{bmatrix} \qquad (V.43)$$

Case II: $n-m-k=0$

$$B_*^{-1} = \begin{bmatrix} B^{-1} & 0 & -B^{-1}R \\ -B^{-1} & I_m & B^{-1}R \\ 0 & 0 & I_k \end{bmatrix} \tag{V.44}$$

We propose Result 4 below.

Result 4: Cases I and II are mutually exclusive.

Proof: Without loss of generality, assume $n > m$, since $n=m$ implies a unique solution and $n < m$ implies redundant or inconsistent constraints. Assume the converse of the proposition. Then it is impossible to construct a basis of dimension $(m+n)$, since the last row of (V.43) or (V.44) would consist of all zeroes, hence the result. QED.

The remaining open question concerns the construction of B_*^{-1} from the optimal basis, D, provided by solution by the UBDS algorithm. Recall that at no time may a variable and its complement appear in the basis simultaneously. Result 5 below provides the connection.

Result 5: B_*^{-1} may be constructed from an UBDS tableau as follows:

1. Form B^{-1} from D^{-1} by multiplying the elements of any row of D^{-1} corresponding to a basic complementary variable by (-1).
2. Form $B^{-1}R$ from UBDS tableau columns corresponding to non-basic complementary variables.
3. The remaining n-k columns are determined according to results 2 and 3.

Proof: (1) is simply the application of step 4 of the UBDS algorithm to obtain the compact basis inverse in terms of the "original" variables $\underline{x}$. (2) is immediate by construction of the partition in Result 2. (3) is obvious by construction of B_*^{-1}.

V.6 Postoptimality Analysis of the Requirements Vector

We may now proceed with the development of the rules for postoptimality analysis of the requirements vector. The general form of the problem to be solved is

$$
\begin{aligned}
&\text{MAX } \underline{cx} \\
&\text{s.t. } A\underline{x} = \underline{b} + \theta\underline{r} \\
&\qquad \underline{L} \leq \underline{x} \leq \underline{U} \qquad\qquad\qquad\qquad\qquad \text{(V.45)} \\
&\qquad x_j \geq 0 \text{ and integer}, \qquad j \in N_1 \\
&\qquad x_j \geq 0 , \qquad\qquad\qquad j \in N_2 .
\end{aligned}
$$

We assume, without loss of generality, that the upper and lower bounds on x_j, $j \in N_1$ are integer. After application of the transformation of variables in (V.32) to remove the lower bounds, where $\underline{y} = \underline{x} - \underline{L}$, we obtain the following system of constraints:

$$
\begin{aligned}
&A\underline{y} = \underline{g} + \theta\underline{r} \\
&y_j \geq 0 \text{ and integer} , \qquad j \in N_1 \qquad\qquad \text{(V.46)} \\
&y_j \geq 0 , \qquad\qquad\qquad j \in N
\end{aligned}
$$

where

$$
\underline{g} = \begin{bmatrix} \underline{b} - A\underline{L} \\ \cdots\cdots \\ \underline{U} - \underline{L} \end{bmatrix} = \begin{bmatrix} \underline{g}_1 \\ \cdots \\ \underline{g}_2 \end{bmatrix} , \qquad \text{(V.47)}
$$

and $\underline{r}$, the vector of alterations is such that

$$
\underline{r} = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_m \\ \cdots \\ \underline{0} \end{bmatrix} \qquad \text{(V.48)}
$$

The parameter θ is chosen such that $\theta \in [0,1]$.

After suitably partitioning the rows of $\underline{g}_2$ as $(\underline{g}_2^1, \underline{g}_2^1, \underline{g}_2^1)'$ to conform with the most general case, (V.42), of B_*^{-1}, the feasibility condition (V.7) is

$$B_*^{-1}(\underline{g}+\theta\underline{r}) \underline{>} \underline{0} \quad , \tag{V.49}$$

which yields the following four relationships:

$$1. \quad (B^{-1}\underline{g}_1 - B^{-1}R\underline{g}_2^3) + \theta B^{-1}\,\underline{r} \underline{>} \underline{0} \tag{V.50.1}$$

$$2. \quad [\underline{g}_2^1 - (B^{-1}\underline{g}_1 - B^{-1}R\underline{g}_2^3)] - \theta B^{-1}\underline{r} \underline{>} \underline{0} \tag{V.50.2}$$

$$3. \quad \underline{g}_2^2 \underline{>} \underline{0} \tag{V.50.3}$$

$$4. \quad \underline{g}_2^3 \underline{>} \underline{0} \quad . \tag{V.50.4}$$

Note that (V.50.3) and (V.50.4) simply require that the upper bounds $\underline{U}$ be greater than or equal to $\underline{L}$. Assuming this to be the case, conditions (V.50.1) and (V.50.2) yield the following rules for determining the critical value of θ:

$$\text{Rule I:} \quad \theta_c^1 = \text{MIN}\left[\frac{(B^{-1}\underline{g}_1)_i - (B^{-1}R\underline{g}_2^3)_i}{-(B^{-1}\underline{r})_i}\right], \ (B^{-1}\underline{r})_i < 0$$

$$\text{Rule II:} \quad \theta_c^2 = \text{MIN}\left[\frac{g_{2_i}^1 - (B^{-1}\underline{g}_1 - B^{-1}R\underline{g}_2^3)_i}{(B^{-1}\underline{r})_i}\right], \ (B^{-1}\underline{r})_i > 0 \tag{V.51}$$

$$\text{Rule III:} \quad \theta_c = \text{MIN}\,(\theta_c^1, \theta_c^2) \ ,$$

where $(\cdot)_i$ denotes the i^{th} element of the vector resulting from the matrix operation in parentheses. Computationally, if for a given θ_k, $\theta_k > \theta_c$ the current basis

is not optimal, so reoptimization by the UBDS algorithm for this value of θ is necessary. For discrete θ_k the process is obviously finite, and for continuous θ it is also finite since there exist only a finite number of bases.

Rules I through III assist us in solving the LP relaxation by postoptimality analysis at any given node. Note that the additional constraints due to separation and branching to be appended to (V.46) may be written as

$$\underline{P} \leq \underline{y} \leq \underline{F} \tag{V.52}$$

where $\underline{P}$ and $\underline{F}$ are the bounds on $\underline{y}$ due to branching. At the problem outset, $\underline{P} = \underline{0}$ and $\underline{F} = \underline{U} - \underline{L}$ for all variables, and remain so throughout the algorithm for the continuous variables. Due to the manner in which separation and branching take place, the bounds on $\underline{y}$ given by (V.52) will always be tight as compared to the bounds implied by (V.47). In order to regain the "upper bounded only" form for (V.46) with (V.52) appended, we make another transformation of variables by taking $\underline{z} = \underline{y} - \underline{P}$ to yield

$$\begin{aligned} &A\underline{z} = \underline{g} + \theta\underline{r} \\ &\underline{0} \leq \underline{z} \leq \underline{Q} = \underline{F} - \underline{P} \\ &z_j \geq 0 \text{ and integer}, \quad j \in N_1 \\ &z_j \geq 0 \qquad\qquad\qquad , \quad j \in N_2 . \end{aligned} \tag{V.53}$$

Note that in "decoding" from $\underline{z}$ to $\underline{y}$, integer $\underline{z}$ $\Longleftrightarrow$ integer $\underline{y}$.

The results of Theorems 4 and 5 given below are useful in performing the postoptimality analysis on the requirements vector. Theorem 4 is an extension of a result due to Nauss [49] for the PMILP (V.27).

Definition: A PMILP of the form (V.27) is monotone if

$$F(S^1) \supseteq F(S^2) \supseteq \ldots \supseteq F(S^{K^*}) \;,$$

where $F(S^k)$ denotes the feasible region implied by the constraint set S^k.

Theorem 4: Given the PMILP (V.27) where $\underline{b}_k = \underline{b} + \theta\underline{r}$, $\theta \in [0,1]$, and if $0 \leq \theta_1 \leq \theta_2 \leq 1$, then the PMILP is monotone and, further, the optimal solution value $z(\theta)$ is a non-increasing function of θ.

Proof: Monotonicity is assured by the choice of $\underline{r}$ to yield more restricted problems as θ increases, thus giving

$$F(S^{\theta_1}) \supseteq F(S^{\theta_2}).$$

The non-increasing nature of $z(\theta)$ is immediately obvious as a result of the monotone property. QED

Note that no stronger result for $z(\theta)$ is forthcoming as in the LP case, since discontinuities will exist, in general, in $z(\theta)$ for certain values of θ corresponding to discrete changes in the integer-restricted variables.

The result of Theorem 5 below provides a method for implicitly solving problems of the PMILP when only one component of $\underline{b}$ is parameterized. That is, in Model I (V.2) we consider the special case

$$\underline{r} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ r_i \\ \vdots \\ 0 \\ 0 \end{bmatrix},$$

where r_i appears in the row corresponding to the element of the requirements vector to be parameterized.

<u>Theorem 5</u>: Let β^*_{jk} be optimal for $K = K_1$. Also, let

$$\sum_j \sum_k \beta^*_{jk} = K_2 \leq K_1 .$$

Then the solution of which β^*_{jk} are components is optimal for all K such that

$$K_2 \leq K \leq K_1 .$$

<u>Proof</u>: Because $K \leq K_1$, any solution feasible for K is also feasible for K_1. Hence optimal for $F(S^{K_1}) \geq$ optimal for $F(S^K)$. But since β^*_{jk} is such that

$$\sum_j \sum_k \beta^*_{jk} = K_2$$

is optimal for K_1, it is true that

optimal for $F(S^{K_1}) \leq$ optimal for $F(S^{K_2}) \leq$ optimal for $F(S^K)$.

Hence,

$$\text{optimal for } F(S^{K_1}) = \text{optimal for } F(S^K)$$

for all $K_2 \leq K \leq K_1$. QED

As was emphasized in our discussion of branch and bound, two essential features are the effectiveness of the upper and lower bounds on z in reducing the scope of the enumeration and the efficiency in solving the LP relaxations. The major thrust of this work has been towards solving the LP relaxations of the several problems under consideration by postoptimality analysis. The Basic Algorithm of Section V.4 is the cornerstone for Algorithm One presented below and Algorithm Two to be given in the next section. Thus, we state only the modifications to the Basic Algorithm, which occur in step 4. We assume that only a finite number of discrete values of $\theta \in [0,1]$ are of interest.

<u>Algorithm One</u> (Postoptimality Analysis of the Requirements Vector)

<u>Step 4'</u>: (LP Solution) At node j, obtain the solution to the initial problem under consideration (indexed by p) by the UBDS algorithm. If $\nexists$ a

feasible solution to $(RP_p)^j$, set $\bar{Z}_j^k = -\infty$, $\forall\, k \geq p$, fathom the node, and go to step 2. Otherwise, set $p = p + 1$.

a) Determine $\hat{\theta}_c$ from Rules I-III. If $\theta_p > \theta_c$ go to (b). Otherwise, update $\underline{x}_B$ using (V.50) and check for possible application of the result of Theorem 5, indexing p as needed. For those problems k for which an optimal solution has been found, set $\bar{Z}_j^k = Z_j^{ok}$. If all problems have been considered, go to step 5. Otherwise, $\theta_p > \theta_c$ and go to (b).

b) Execute UBDS pivots to restore primal feasibility. If this cannot be done, set $\bar{Z}_j^k = -\infty$, $\forall\, k \geq p$, fathom the node, go to step 2. If feasibility is restored, go to (a).

Algorithm One was coded by the author in FORTRAN IV, incorporating the features discussed in this section. Problems of the general nature of Model I (V.2) were randomly generated (e.g., A, $\underline{b}$, $\underline{c}$, $\underline{L}$, and $\underline{U}$), with the exception that A did not have the special structure or sparsity as in (V.2). Table V.1 indicates some sample results. Note that we have indicated the number of constraints (m), total number of variables (n), the matrix

Table V.1
Results of Computational Study for Algorithm One

Problem	m	n	Density %	% Var. $\bar{c}$ UB=∞	# Sol'ns. Req'd.	# Int. Vars.	Time (Min.)	LP's	Vertices	Postopt. Calls
1.	6	13	90	0	6	5	.09	88	31	23
						6	.09	96	35	26
						7	.10	100	39	27
2.	10	30	60	10	6	5	.02	6	1	1
						10	.19	46	13	13
						15**	3.67+	780	283	---
3.	10	30	60	10	6	5	.41	87	21	21
						10	1.38	286	69	64
						15	9.01	1,572	457	387
						20**	8+	1,548	463	---
4.	10	30	60	10	6	5	.14	39	9	8
						10	.90	217	69	58
5.	15	30	50	10	6	5	1.11	81	23	17
						10	10.08	595	187	152
						15*	7.71+	464	179	---
6.	15	30	50	10	6	5	.51	65	19	18
						10	1.90	219	75	64
7.	15	30	50	10	6	5	.28	43	13	13
						10	1.24	182	69	58

* Termination due to time limitations; integer feasible solution not available.

** Termination due to time limitations; integer feasible solution available.

density, number of integer-restricted variables, percentage of variables with large upper bounds, and the number of PMILP problems to be solved. All of these obviously affect the performance of the algorithm. Also reported are CPU time IBM(7094), number of nodes in the enumeration tree, the number of LPs solved (including solution from scratch and pivoting to restore feasibility), and the number of times the postoptimality analysis routine was called into use.

V.7 Postoptimality Analysis of the Upper and Lower Bounds

Turning to the postoptimality analysis of upper and lower bounds, we state the following general problem form:

$$\begin{aligned} &\text{MAX } \underline{cx} \\ &\text{s.t. } A\underline{x} = \underline{b} \\ &\underline{L} + \theta(\underline{L}^* - \underline{L}) \leq \underline{x} \leq \underline{U} + \alpha(\underline{U}^* - \underline{U}) \qquad \text{(V.54)} \\ &x_j \geq 0 \text{ and integer}, \quad j \in N_1 \\ &x_j \geq 0, \quad j \in N_2 \end{aligned}$$

As before, we wish to transform (V.54) into an "upper bounded only" form in the variables $\underline{y}$. After doing so, we have the following system of constraints:

$$A\underline{y} = \underline{g} + \alpha\,\underline{d}_1 - \theta\underline{d}_2 \qquad \text{(V.55)}$$

$$y_j \geq 0 \text{ and integer }, \quad j \in N_1$$

$$y_j \geq 0 \qquad , \quad j \in N_2 ,$$

where

$$\underline{g} = \begin{bmatrix} \underline{b} - A\underline{L} \\ ---- \\ (\underline{U} - \underline{L}) \end{bmatrix} = \begin{bmatrix} \underline{g}_1 \\ ---- \\ \underline{g}_2 \end{bmatrix} \qquad \text{(V.56)}$$

$$\underline{d}_1 = \begin{bmatrix} \underline{0} \\ ---- \\ \underline{U}^* - \underline{U} \end{bmatrix} = \begin{bmatrix} \underline{d}_1^1 \\ ---- \\ \underline{d}_1^2 \end{bmatrix} \qquad \text{(V.57)}$$

and

$$\underline{d}_2 = \begin{bmatrix} A(\underline{L}^* - \underline{L}) \\ ------- \\ \underline{L}^* - \underline{L} \end{bmatrix} = \begin{bmatrix} \underline{d}_2^1 \\ ---- \\ \underline{d}_2^2 \end{bmatrix} \qquad \text{(V.58)}$$

The upper and lower partitions of $\underline{g}$, $\underline{d}_1$, and $\underline{d}_2$ consist of m and n rows, respectively. Note that this is the same representation of the parameterization as given in Model II (V.3).

Recall that in Section V.2 we discussed the problems inherent in simultaneous parameterization for two parameters. These problems are certainly of concern here. In addition, however, we must consider the problem of transforming a solution in $\underline{y}$ to one in $\underline{x}$ for the

integer-restricted variables. We cannot, in general assume that the lower bound on such variables will be integral, due to the parametric nature of the bound with respect to θ. Further, as will be seen below, the constraints added to (V.55) due to branching will also be transformed by lower bounds, but must be integer. Thus, it is first convenient to fix θ a priori at $\theta = \bar{\theta}$. Note that such an approach requires the solution of as many PMILP as there are discrete values of θ. Next, for $j \in N_1$, in order that integer $\underline{x} \Leftrightarrow$ integer $\underline{y}$, for those values of $\bar{\theta}$ and j such that $L_j^* + \theta(L_j^* - L_j)$ is not integer. the lower bound constraint,

$$[L_j + \bar{\theta}(L_j^* - L_j)] + 1 \leq x_j \ , \quad j \in N_1$$

is substituted. Because of this, the right-hand side of (V.55) is no longer a linear function of θ. We instead may write (V.55) as

$$\begin{aligned} A\underline{y} &= \underline{g} + \alpha \underline{d}_1 - \underline{w}(\theta) & & \\ y_j &\geq 0 \text{ and integer}, & j &\in N_1 \qquad \text{(V.59)} \\ y_j &\geq 0 \ , & j &\in N_2 \ .^{**} \end{aligned}$$

Proceeding as before, using the form (V.42) of B_*^{-1}, the feasibility condition for the expanded system of constraints,

$$\underline{y}_B = B_*^{-1}(\underline{g} + \alpha \underline{d}_1 - \underline{w}(\theta)) \geq \underline{0} \qquad \text{(V.60)}$$

yields the following four relationships. Note that

**The vector $\underline{w}(\theta)$ is formed from $\underline{d}_2$ by appropriately modifying those elements of $\underline{d}_2$ affected by the substitution of lower bounds.

$\underline{g}_2$, $\underline{d}_1^2$, and $\underline{w}_2(\theta)$ have been partitioned to conform with the partition of B_*^{-1} as

$$\underline{g}_2 = \begin{bmatrix} g_2^1 \\ g_2^2 \\ g_2^3 \end{bmatrix}, \quad \underline{d}_1^2 = \begin{bmatrix} \underline{d}_1^{21} \\ \underline{d}_1^{22} \\ \underline{d}_1^{23} \end{bmatrix}, \quad \underline{w}_2(\theta) = \begin{bmatrix} \underline{w}_2^1(\theta) \\ \underline{w}_2^2(\theta) \\ \underline{w}_2^3(\theta) \end{bmatrix}$$

1. $(B^{-1}\underline{g}_1 - B^{-1}R\underline{g}_2^3) - (\alpha B^{-1}R\underline{d}_1^{23} + B^{-1}\underline{w}_1(\theta) - B^{-1}R\ \underline{w}_2{}^3(\theta)) \geq \underline{0}$

2. $(\underline{g}_2^1 - B^{-1}\underline{g}_1 + B^{-1}R\underline{g}_2^3) + (\alpha B^{-1}R\underline{d}_1^{23} + B^{-1}\underline{w}_1(\theta) - B^{-1}R\ \underline{w}_2^3(\theta)) + \alpha\ \underline{d}_1^{21}$

   $- \underline{w}_2^1(\theta) \geq \underline{0}$ (V.61)

3. $\underline{g}_2^2 + \alpha\ \underline{d}_1^{22} - \underline{w}_2^2(\theta) \geq \underline{0}$

4. $\underline{g}_2^3 + \alpha\ \underline{d}_1^{23} - \underline{w}_2^3(\theta) \geq \underline{0}$

Based on the relationships (V.61), we may define the following rules for determining critical values of α.

Rule I:

$$\alpha_c^1 = \text{MIN}\left[\frac{(B^{-1}\underline{g}_1)_i - (B^{-1}R\underline{g}_2^3)_i + (B^{-1}\underline{w}_1(\theta))_i - (B^{-1}R\underline{w}_2^3(\theta))_i}{(B^{-1}R\underline{d}_1^{23})_i}\right]$$

for $(B^{-1}R\underline{d}_1^{23})_i > 0$

Rule II:

$$\alpha_c^2 = \text{MIN}\left[\frac{g^1_{2_i} - (B^{-1}\underline{g}_1)_i + (B^{-1}R\underline{g}^3_2)_i + (B^{-1}\underline{w}_1(\theta))_i - (B^{-1}R\underline{w}^3_2(\theta))_i - w^1_{2_i}(\theta)}{-[(B^{-1}R\underline{d}^{23}_1) + \underline{d}^{21}_1]_i}\right]$$

$$\text{for } [(B^{-1}R\underline{d}^{23}_1 + \underline{d}^{21}_1]_i < 0$$

Rule III:

$$\alpha_c^3 = \text{MIN}\left[\frac{g^2_{2_i} - w^2_{2_i}(\theta)}{-d^{22}_{1_i}}\right], \text{ for } d^{22}_{1_i} < 0$$

Rule IV:

$$\alpha_c^4 = \text{MIN}\left[\frac{g^3_{2_i} - w^3_{2_i}(\theta)}{-d^{23}_{1_i}}\right], \text{ for } d^{23}_{1_i} < 0$$

Rule V:

$$\alpha_c = \text{MIN}\ (\alpha_c^1, \alpha_c^2, \alpha_c^3, \alpha_c^4)$$

The critical value α_c obtained in Rule V is that value of α, given $\theta = \bar{\theta}$, such that the current optimal basis will become infeasible, necessitating a change in the basic variables. Again, the critical values may be used for comparison with predetermined discrete α or to find all feasible bases of the problem. By an argument similar to that given in connection with parameterization of the requirements vector, the process is finite.

When imbedding the postoptimality analysis technique for upper and lower bounds into the Basic Algorithm,

the process is not as straightforward as in the case of the requirements vector. This is primarily because of the intimate connection between the process of separation and branching and the parameterization involved. That is, the set of branching constraints may dominate or be dominated by the parameterized bound restrictions, the changeover often occurring within the scope of the parameterization at the node. In order to examine this phenomenon more closely, we begin by considering the system (V.59) which, in this case, has already had the lower bound transformation applied. We thus have

$$A\underline{y} = \underline{g} + \alpha\,\underline{d}_1 - \underline{w}(\theta) \qquad \text{(V.62)}$$

$$\underline{P} \leq \underline{y} \leq \underline{F}$$

$$y_j \geq 0 \text{ and integer}, \qquad j \in N_1$$

$$y_j \geq 0 \qquad\qquad, \qquad j \in N_2 \quad .$$

There are two central issues to be examined with respect to the parametric bounding constraints and additional branching constraints:

1. Consistency between the parametric upper bounding constraints and lower bound constraints due to branching. (If they are inconsistent, this implies infeasibility.)
2. Dominance of either the parametric upper bound constraints or the upper bound

constraints due to branching, the latter implying non-parametricity of the upper bound over the range of α for which the dominance holds.

Without loss of generality, we chose to examine the applicable portion of constraint set (V.62)

$$\begin{aligned} &\underline{0} \leq \underline{y} \leq \underline{g}_2 + \alpha\, \underline{d}_1 - \underline{w}_2(\theta) \\ &\underline{P} \leq \underline{y} \leq \underline{F} \\ &y_j \geq 0 \text{ and integer,} \qquad j \in N_1 \\ &y_j \geq 0 \qquad\qquad\qquad , \qquad j \in N_2 \;. \end{aligned} \tag{V.63}$$

With θ fixed at $\bar{\theta}$, we may derive the following limits on α.

1. Feasibility/Consistency:

$$\underline{P} \leq \underline{g}_2 + \alpha\, \underline{d}_1^2 - \underline{w}_2(\theta)$$

must hold for consistency. In general, $\underline{d}_1^2 \leq \underline{0}$ since we consider more restricted problems as α increases, so that the value of α beyond which the problem would be infeasible is given by

$$\alpha_1 = \underset{j \in N_1}{\mathrm{MIN}} \left[\frac{P_j - g_{2_j} + w_{2_j}(\theta)}{d_{1_j}^2} \right], \text{ for } d_{1_j}^2 \neq 0 \;. \tag{V.64}$$

For the dominance question, when the parametric upper bound is applicable we say the bound is "released." When the fixed upper bound due to branching is tight, we say

[illegible]

the bound is "pegged."

II. Pegging/Releasing of Integer-Restricted Variables as a Function of α.

The branching constraints will be tight if

$$\underline{D} \le \underline{g}_2 + \alpha \underline{d}_1 - \underline{w}_2(\theta)$$

Thus, the upper bounds on all integer-restricted variables will be pegged for those $\alpha \le \alpha_2$, where

$$\alpha_2 = \min_{j \in N_1} \left[\frac{D_j - g_{2_j} + w_{2_j}(\theta)}{d^2_{1_j}} \right], \quad \text{for } d^2_{1_j} \ne 0 . \qquad \text{(V.65)}$$

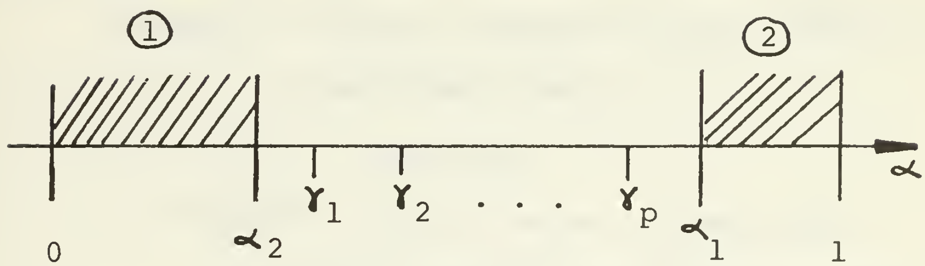
Further, for $\alpha_2 \le \alpha \le \alpha_1$, each integer variable will have its upper bounds again made parametric for all $\alpha \ge \gamma_j$, where γ_j is given by

$$\gamma_j = \left[\frac{D_j - g_{2_j} + w_{2_j}(\theta)}{d^2_{1_j}} \right], \quad \text{for } j \in N_1 \text{ and } d^2_{1_j} \ne 0 .$$

We call the γ_j "release points." The process defined by I and II above may be visualized as in Figure V.1, where 1 is the range of α for which all the upper bounds on integer-restricted variables are pegged, and 2 is the range of α for which the problem is infeasible. We assume the γ_j have been rank ordered. Note that they are not necessarily distinct and not necessarily positive. An additional result is possible for the ILP, namely an

Figure V.1

Illustration of Consistency/Feasibility and Pegging/Releasing Rules



implicit solution routine. That is, for all $\alpha \leq \alpha_2$ the problem is no longer parametric on α, so it is sufficient to solve the problem at $\alpha = \alpha_2$ and by implication determine solutions for $\alpha \leq \alpha_2$. We demonstrate two additional results as Lemmas 1 and 2 for the rules proposed above.

<u>Lemma 1</u>: For the feasibility and pegging scheme described above,

$$\alpha_2 \leq \alpha_1.$$

Proof: Recall that α_1 is such that

$$P_j = g_{2_j} + \alpha_1 d^2_{1_j} - w_{2_j}(\theta), \text{ for some } j \in N_1$$

and α_2 is such that

$$D_j = g_{2_j} + \alpha_2 d^2_{1_j} - w_{2_j}(\theta), \text{ for some } j \in N_1$$

Thus, $P_j \leq D_j \; \forall \; j$ and $d^2_{1_j} \leq 0$ yield the result.

QED

<u>Lemma 2</u>: If $d^2_{1_j} = 0$, then $\gamma_j = \infty$ and the upper bound due to the branching restrictions is tight $\forall \alpha$.

<u>Proof</u>: Immediate, since the assumption implies non-parametricity.

Finally, we would like the expanded system (V.62) to be in "upper bounded only" form to facilitate solution by the UBDS algorithm. Therefore, we apply the transformation of variables

$$\underline{z} = \underline{y} - \underline{P} \tag{V.66}$$

to yield

$$A\underline{z} = \underline{g} + \alpha\,\underline{d}_1 - \underline{w}(\theta) - A\underline{P} \tag{V.67}$$

$$\underline{0} \leq \underline{z} \leq \underline{F} - \underline{P}$$

$$z_j \geq 0 \text{ and integer}\,, \qquad j \in N_1$$

$$z_j \geq 0 \qquad\qquad\qquad , \qquad j \in N_2\ .$$

Note, however, that if we were to examine the constraints implied by the lower partition of $\underline{g}$, $\underline{d}_1$, and $\underline{w}(\theta)$ we would have

$$-\underline{P} \leq \underline{z} \leq \underline{g}_2 + \alpha\,\underline{d}_1 - \underline{w}_2(\theta) \tag{V.68}$$

But since $\underline{P}$ is necessarily non-negative, being a vector of lower bounds on necessarily non-negative variables, the left-hand inequality of (V.68) is dominated by $\underline{z} \geq \underline{0}$, and hence, (V.67) is the correct general form. The objective functions corresponding to the constraint sets (V.62) and (V.67) are altered by constant terms which do not affect the optimization.

As was the case in Section V.6, the developments of this section involve the modification of step 4 of the Basic Algorithm. Hence, we present the new step below, the remainder of Algorithm Two being the same as the Basic Algorithm. It is assumed that only a finite number of discrete values of $\alpha \in [0,1]$ are of interest, these being indexed by 1,2,...,K*.

[illegible]

<u>Algorithm Two</u>: (Postoptimality Analysis of the Upper and Lower Bounds)

<u>Step 4'</u>: (LP Solution) At node j, obtain the solution to the initial problem under consideration (indexed by p, implying α_p and $\bar{\theta}$ are operable) from the UBDS algorithm. If $\nexists$ a feasible solution for $(RP_p)^j$, set $\bar{z}_j^k = -\infty$, $\forall\, k \geq p$, fathom the node, and go to step 2. Otherwise, p = p + 1 and proceed as follows:

a) Transform the problem in $\underline{x}$ to one in $\underline{z}$ with appropriate transformations in $\underline{P}$ and $\underline{F}$. Take care to insure integrality of lower bounds for integer-restricted variables.

b) Compute α_1, α_2, and γ_j, $j \in N_1$.

c) If p = K* fathom the node and go to step 5. If $\alpha_p > \alpha_1$, set $\bar{z}_j^k = -\infty$, $\forall\, k \geq p$, fathom the node, and go to step 2.

d) Otherwise update $\underline{x}_B$ using the results of (V.61), modified by the transform $\underline{z} = \underline{y} - \underline{P}$, choosing the proper form for the problem upper bound in accordance with γ_j, $j \in N_1$ (e.g., either $g_{2_j} + \alpha\, d_{1_j}^2 - w_{2_j}(\theta)$ or $D_j - P_j$, the latter case requiring redefinition of $\underline{g}_2$, $\underline{d}_1^2$, and $\underline{w}_2(\theta)$.

e) After updating, if primal feasibility exists, set $\bar{z}_j^k = z_j^{op}$, p = p + 1, go to (c). Otherwise,

execute UBDS pivots to regain feasibility. If possible, set $p \rightarrow p + 1$, $\bar{Z}_j^k = Z_j^{op}$ and go to (c). If not possible, set $\bar{Z}_j^k = -\infty$, $\forall\, k \geq p$, fathom the node, and go to step 2.

(Note: If desired, a transformation from $\underline{z}$ to $\underline{x}$ may be made in (e).)

Algorithm Two was coded in FORTRAN IV and run for several randomly generated problems. In addition to the other randomly generated model components, the vectors $\underline{L}^*$ and $\underline{U}^*$ were also generated. The results of the computational study are presented in Table V.2.

V.8 Conclusion

In this chapter, we have developed and proposed methodologies for the efficient solution of the class of parametric mixed-integer linear programs typified by Model I and Model II. From a computational point of view, further improvements in algorithmic efficiency based on the special structures of (V.2) and (V.3) will be discussed in Chapter VI.

One of the more interesting issues raised in this chapter concerns PMILP which are simultaneously parametric on two parameters. In Section V.7, we indicated how the bivariate case could be reduced to a univariate case by fixing θ *a priori* and solving the PMILP for the values of

Table V.2
Results of Computational Study for Algorithm Two

Problem	m	n	Density %	% Var. UB=∞	α Step Size	Θ Step Size	# Int. Vars.	Time (Min.)	LP's	Vertices	Postopt. Calls
1.	6	13	90	0	.2	1.0	5	.12	39	17	12
2.	10	25	50	10	.25	1.0	5	1.50	116	39	35
							10	6.13	546	199	147
							15	11.37	926	375	263
3.	10	25	60	0	.25	1.0	5	.45	28	17	13
							10	5.81	312	179	143
4.	10	25	50	10	.25	1.0	5	.98	113	47	45
							10	5.47	518	299	259
5.	10	30	60	10	.2	1.0	5	.18	6	5	4
							10	3.26	138	71	51
							15	4.43	201	95	68
							20*	10.5+	506	251	---
6.	10	30	60	0	.2	1.0	5	1.11	96	47	38
							10	5.42	422	239	179
7.	10	30	60	10	.2	1.0	5	.15	14	7	4
							10	1.60	96	57	43

* Termination due to time limitation; integer feasible solution available.

α under consideration. When all values of α have been considered, θ may then be fixed at another value of interest, the algorithm reapplied, and so on. This essentially requires solving one PMILP problem for each value of θ of interest.

Recall, however, that in addition to fixing θ to reduce the problem to the univariate case, further modifications to the lower bounds on integer-restricted variables for a given $\bar{\theta}$ were necessary so that integer $\underline{x} \Leftrightarrow$ integer $\underline{y}$. It is proposed that if there existed a dual LP algorithm that did not require a transformation of variables to remove non-zero lower bounds, but instead handled the lower bounds by a scheme analogous to that for the upper bounds, the following strategies might be employed.

First, at any node we could obtain information relative to all LP relaxations implied by the combinations of α and θ by fixing θ at the first value of interest and solving the LP relaxations for all α. The parameter θ could then be fixed at its next value, LP relaxations obtained for all α, and so on. This process may be visualized on the unit rectangle as proceeding on the line from $\alpha=0$ to $\alpha=1$ for each fixed θ, and is called the "line approach." Figure V.2 illustrates the process. Note that in this case, an additional set of

conditions for checking the consistency of the parametric lower bounds with the upper bounds due to branching and a pegging/releasing rule for θ would be useful. Care would have to be taken, however, that the feasibility conditions derived for α and θ were applied for the proper combinations of values of these parameters. Although the line approach would give a "one-pass" algorithm as opposed to the repeated application of Algorithm Two, it is not intuitively clear that the gain in computational efficiency would be substantial. This is due mainly to the relative appropriateness for the several combinations of α and θ of the branchings undertaken.

A second approach that might be useful at a node fixes θ at its initial value and solves the LP relaxations for all values of α of interest. The parameter α is then fixed at its maximum value and LP relaxations obtained for all θ of interest. Then, fixing θ at its final value not previously encountered, consideration could be shifted to α, and so on. Note that we would never have to reconsider combinations of α and θ already examined. For this reason, this process is called the "spiral approach," and is illustrated in Figure V.3. The principal merit of the spiral approach

[illegible]

[illegible]

is that when we begin parameterization with that parameter previously held constant, an optimal basis is immediately available. This is not the case in the line approach, where the problem with a new value of θ must be started from scratch. As was the case in the line approach, conditions on consistency and pegging/releasing would have to be made available for both parameters, again with the caveat on their proper application. In addition, a set of postoptimization rules analogous to those in Section 7 for α would be necessary for θ. The additional bookkeeping required in the spiral approach would appear to be a significant consideration.

The development of the upper and lower bounded dual simplex algorithm and its incorporation into PMILP algorithms based on the line and spiral approaches appears to be an area for further research. Comparison of computational experience from such methods with that obtained from our proposed methodology would be useful. Finally, a direct attempt at using Algorithm Two in a line approach might prove to be an interesting application.

Figure V.2

Line Approach to Reduction of Bivariate Parameterization

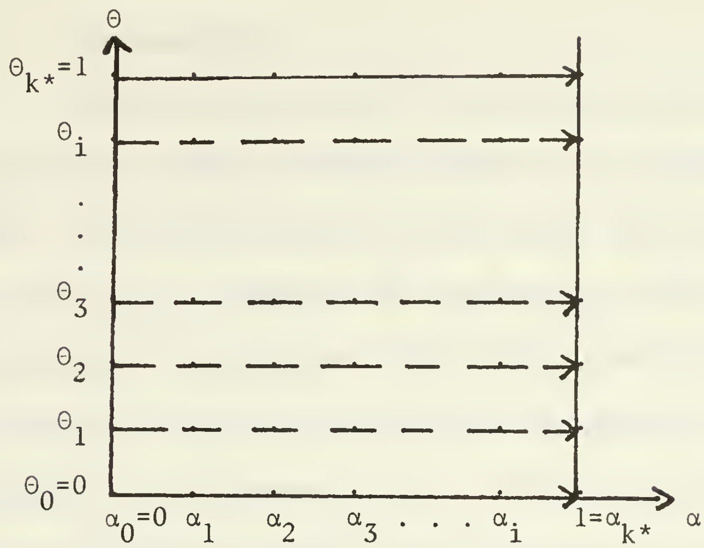
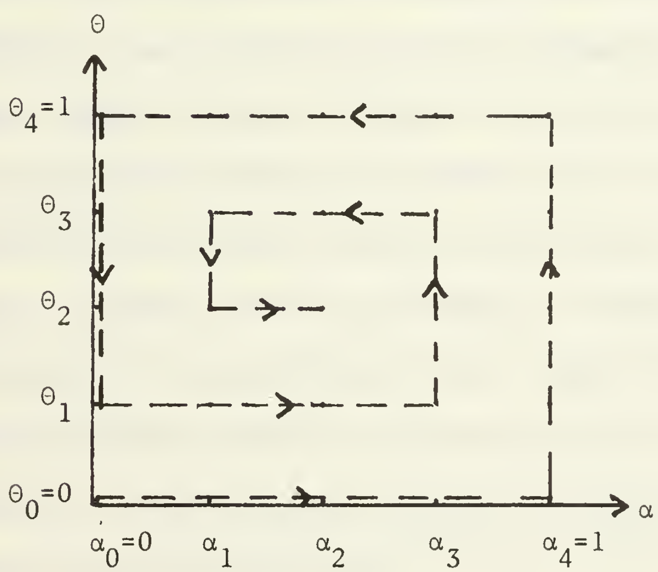


Figure V.3

Spiral Approach to Reduction of Bivariate Parameterization
(Illustrated for K* = 5)



Chapter VI. An Application of the Methodology

VI.1 Introduction

Having developed a patient classification system for level of care required, the Basic Staffing Model, and extensions to the Basic Staffing Model, we turn to an example of the way in which the methodology can be applied. We shall demonstrate the efficacy of Models I and II for use in determining staffing patterns, the computational characteristics of algorithms for each model having been previously shown. As a basis for the application, the nurse staffing problem and the underlying assumptions will be delineated, along with the necessary data elements. Some computational considerations for problems of the form of Models I and II will also be discussed. Following this, applications of Model I will be presented, with consideration devoted to variation in the total service level and changes in personnel budget restrictions. Model II will be used to demonstrate the effects of alterations in the upper and lower bounds of care activity times on nursing service requirements. Finally, comments are offered on policy implications for institutional care.

VI.2 Problem Construction and Computational Considerations

As the setting for our example, we have selected a representative proprietary long-term care facility in the Baltimore metropolitan area. This facility maintains 100 licensed beds for both Intermediate A and Skilled Nursing Care patients, and is generally recognized by professionals as providing good quality care. A facility with two levels of care, such as this one, is typical; only the much larger and more complex facilities have three or perhaps four levels, including chronic care. Note that the consideration of two levels of care instead of the three basic levels for which our models have been designed can be accomplished quite easily; such a problem is just a subset of the larger problem. Initially, we assume a patient mix of 50 Intermediate A care patients and 50 Skilled Nursing Care patient as determined by the classification system of Chapter IV.

Recall that in Chapter III certain data necessary for the specification of the staffing models were omitted. These include:

1. Upper and lower bound restrictions on nursing staff by skill level;
2. Salary costs by skill level;
3. Personnel budget allowance.

Since we have previously indicated that the methodology is best applied to the daily (24-hour) staffing problem,

items (1), (2), and (3) will be presented within this context.

In setting the bounding restrictions on nursing staff, there are two major considerations. First, sufficient staff must be on duty each day to satisfy state regulations. Obviously, this constraint may be incorporated into the lower bounds. On the other hand, the exigencies of the local labor market for nursing personnel, especially licensed (R.N. and L.P.N.) nurses, set an upper bound on the number of nurses of each skill level that can be hired. Of course, budgetary restrictions may enforce a staff mix that is considerably below the upper bounds set in this manner.

In order to set lower bounds, nursing home regulations for the State of Maryland were consulted.* These regulations specify, as a minimum, that for both Intermediate A and Skilled Care patients the ability to provide two hours of bedside care per patient day must be maintained. In addition, a staff-to-patient ratio of 1:25, again as a minimum, must be adhered to. Finally, for the assumed mix of Intermediate A and Skilled Care patients, two R.N.'s and three L.P.N.'s must be on duty in a supervisory/charge and care-providing capacity. Additional regulations concerning

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* Maryland State Department of Health Regulations 43G02, 43G06, and 43G07.

[illegible]

licensed nurse coverage by shift are provided, but are not essential to the models. Taking all of these factors into consideration, upper and lower bounds on nursing staff were set as shown in Table VI.1. It should be noted that the minimum mix represents a least-cost feasible solution.

Table VI.1

Upper and Lower Bounds on Nursing Personnel for the Example

	R.N.	L.P.N.	N.A.
Upper Bound	8	20	40
Lower Bound	2	3	23

In computing the values in Table VI.1 a 7 1/2-hour shift was used as the basis for determining compliance with minimum daily bedside care requirements.

With respect to daily staffing costs, data were obtained from the facility under study as to costs for licensed and non-licensed nurses averaged over all three shifts, including fringe benefits. After application of an adjustment for the salary differential between R.N.'s and L.P.N.'s, the average daily staffing salary costs for each skill level were determined to be as shown in Table VI.2.

Table VI.2

Average Daily Salary Cost of Nursing Personnel, by Skill Level, for the Example

R.N.	L.P.N.	N.A.
$46.88	$39.75	$27.15

The current average daily nursing personnel budget for the facility was determined to be approximately $885.00.

Because of the use of a 7 1/2-hour shift in our example facility, the results of Table III.8 concerning nursing time availability for patient-centered activities per shift must be modified. The 7 1/2-hour shift is derived by subtracting one-half hour for meals from the 8-hour shift. This being the case, the percentage allowance for meals was removed from the 8-hour shift data to give the percentages of time, per shift, devoted to non-patient-centered activities. By applying these percentages to the 7 1/2-hour shift, the availabilities in minutes as shown in Table VI.3 were obtained.

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]

[illegible]

[illegible]

Table VI.3

Recomputed Availabilities, per 7 1/2-Hour Shift, in Minutes, by Nursing Skill Level

Skill Level	Percentage	Availability Per 7 1/2-Hours
R.N.	82.4	370.8
L.P.N.	84.6	380.7
N.A.	69.1	311.0

With the remainder of the data for the problem as described in Chapter III, the models are thus specified. Before beginning the application, however, certain computational features of Models I and II are worthy of mention. Recall that in the description of the Upper Bounded Dual Simplex algorithm in Chapter V, no specific computational strategies were specified. The implication was that the algorithm would be executed on a complete simplex-type tableau. Examination of the constraint sets of Models I and II reveals, however, that the corresponding constraint matrix contains quite a large number of zeroes; that is, it is sparse. Significant improvements in algorithmic efficiency are possible in such cases, due mainly to decreased computer storage requirements and numbers of calculations. By storing non-zero constraint entries in lists and using an upper-bounded dual form

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of the revised simplex method (cf. Hadley [28]), the requirement for maintenance of a complete tableau is eliminated, thus improving efficiency. These aspects were incorporated into Algorithms One and Two when solving the example problems of this chapter.

VI.3 Example of Model I for Service Level Variation

We turn now to the application of Model I (III.2) to our example problem. In this section we will consider the effect on the objective function and staff mix of variations in the total number of minutes of patient-centered activities provided, S. Such a study assists the administrator in determining the sensitivity of the proposed model to varying amounts of service provided. By fixing the personnel budget at any given level, a determination can be made as to how much service such a budget would support, and where possible improvements could be made vis-a-vis staff mix and assignment.

For our purposes, we have chosen to set the daily budget at a high level ($2000.00) to yield an indication of the ideal staff mix to provide any given level of service. Three proposed patient mixes, by classification, are examined: 50 Int. A and 50 Skilled, 40 Int. A and 60 Skilled, and 60 Int. A and 40 Skilled patients. The range of S for consideration was chosen

[illegible]

[illegible]

[illegible]

[illegible]

in each case to approximate the difference between the maximum and minimum service levels obtained by summing the upper and lower bounds, respectively, for each patient mix. For the sake of brevity, we present results in tabular form, omitting detailed specification of the assignment patterns. Tables VI.4 through VI.6 display the information, where Z indicates the objective function and S is constrained to be less than or equal to the value indicated.

Table VI.4

Model I Solution for Various Service Levels for a Patient Mix of 50 Int. A and 50 Skilled Patients

Service Level (S) (mins.)	Z	Staff Mix* RN, LPN, NA			$ to Support Staff Mix
16500	32419.4	8	19	20	1673.29
15720	32419.4	8	19	20	1673.29
14940	31395.4	8	17	19	1566.64
14160	29835.4	8	15	18	1459.99
13380	28275.4	8	14	18	1420.24
12600	26384.4	8	13	16	1326.19

* Actual staffing; the lower bound of 23 for NA given in Table VI.1 may be relaxed due to sufficient staff to meet regulations.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Table VI.5

Model I Solution for Various Service Levels for a Patient Mix of 40 Int. A and 60 Skilled Patients

Service Level(S) (mins.)	Z	Staff Mix* RN,LPN,NA	$ to Support Staff Mix
17400	32174.4	8 20 20	1713.04
16600	32174.4	8 20 20	1713.04
15800	31574.6	8 20 18	1658.74
15000	29974.6	8 18 18	1579.24
14200	28374.6	8 16 17	1472.59
13400	26483.6	8 15 17	1432.84

*Actual staffing; the lower bound of 23 for NA given in Table VI.1 may be relaxed due to sufficient staff to meet regulations.

Table VI.6

Model I Solution for Various Service Levels for a Patient Mix of 60 Int. A and 40 Skilled Patients

Service Level(S) (mins.)	Z	Staff Mix* RN,LPN,NA	$ to Support Staff Mix
15500	31734.4	8 18 16	1524.94
14560	31370.4	8 15 20	1514.29
13620	29796.2	8 13 19	1407.64
12680	27916.2	8 11 19	1328.14
11740 and below--no feasible solution			

*Actual staffing; the lower bound of 23 for NA given in Table VI.1 may be relaxed due to sufficient staff to meet regulations.

As might be anticipated, the cost of providing care for a patient mix requiring predominantly Skilled care is uniformly greater than either of the costs for other patient mixes examined. In addition, there

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

is greater reliance on licensed nurses for care provision in this case than for either of the other mixes. The normative nature of the model and data, as well as the relaxed budget constraint, are primarily responsible for the heavy reliance on R.N.'s in all three configurations. Two final points are of interest. In all three cases, the largest increase in the objective function per dollar expended occurs at the lower end of the spectrum. At the upper end of the service level range for the 50-50 and 40-60 patient mixes, further increases in Z are not forthcoming beyond the S values of 15720 minutes and 16600 minutes, respectively. The indication here is that the true upper bound on S lies somewhere between the two largest values and/or the model is being forced into making assignments for which the appropriateness scores are zero.

VI.4 Example of Model I for Budget Variation

While the examination of changes in the optimal solution to the BSM as a function of the service level via Model I is, perhaps, more of academic interest, the use of Model I for testing the effects of alterations in the personnel budget can be of great value to an administrator. In this regard, Model I was applied for the case of a patient mix of 50 Int. A

and 50 Skilled care patients. The budget was chosen to range from $1200 to $1800 per day, in steps of $200. This range would include feasible staff mixes for both the maximum and minimum total service that can be provided under the upper and lower bounds in use.

Results of this application are shown in detail in Figure VI.1. The output not only shows the objective function, staff mix, and budget constraint, but also the optimal assignment pattern by care area, nursing skill level, and patient classification (A=Int. A and S=Skilled care). These results are summarized in Table VI.7 below.

Table VI.7

Model I Solution for Various Budget Levels for a Patient Mix of 50 Int. A and 50 Skilled Patients

Budget($)	Total Service Provided(min.)	Z	Staff Mix RN,LPN,NA	$ to Support Staff Mix
1800	16194	32419.4	8 19 24	1781.89
1600	15829.9	31156.9	8 15 23	1595.74
1400	13955.23	28027.4	8 10 23	1396.99
1200	12463	21404.6	2 12 23	1195.21

Several comments on this result are in order. First, the increase in the budget from $1200 to $1400 is obviously of high significance, as it precipatates a shift in the staff mix that allows a large increase

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Figure VI.1

Results of Application of Model I for Budget Variation

FINAL REPORT

THIS PROBLEM CONTAINED 51CONSTRAINTS(5.LE. AND 46.GE.) THERE WERE 187VARIABLES, 3OF WHICH WERE INTEGER.

RHSK RANGES FROM 1800.0TO 1200.0IN STEPS OF -200.0

BUDGET= $ 0.00

RHSK= 1800.0 Z(K)= 32419.4

STAFF MIX: RN: 8, LPN:19, NA:24,

STAFF/PT. CLASS

	RN		LPN		NA		TOTAL	
	A	S	A	S	A	S	A	S
AREA 1	0.0	0.0	0.0	475.0	275.0	0.0	275.0	475.0
AREA 2	75.0	350.0	0.0	0.0	0.0	0.0	75.0	350.0
AREA 3	0.0	1000.0	300.0	1000.0	1000.0	250.0	1300.0	2250.0
AREA 4	0.0	0.0	0.0	850.0	450.0	0.0	450.0	850.0
AREA 5	0.0	0.0	0.0	425.0	375.0	0.0	375.0	425.0
AREA 6	0.0	0.0	137.5	750.0	0.0	0.0	137.5	750.0
AREA 7	0.0	0.0	0.0	0.0	200.0	250.0	200.0	250.0
AREA 8	0.0	0.0	0.0	125.0	50.0	0.0	50.0	125.0
AREA 9	0.0	0.0	22.5	37.5	0.0	0.0	22.5	37.5
AREA 10	0.0	0.0	0.0	500.0	375.0	0.0	375.0	500.0
AREA 11	0.0	0.0	0.0	1000.0	1000.0	1000.0	1000.0	2000.0
AREA 12	0.0	501.4	500.0	198.6	0.0	0.0	500.0	700.0
AREA 13	0.0	0.0	87.5	225.0	0.0	0.0	87.5	225.0
AREA 14	37.5	150.0	0.0	0.0	0.0	0.0	37.5	150.0
AREA 15	0.0	0.0	37.5	25.0	0.0	0.0	37.5	25.0
AREA 16	0.0	0.0	0.0	100.0	75.0	0.0	75.0	100.0
AREA 17	0.0	0.0	212.5	212.5	0.0	0.0	212.5	212.5
AREA 18	362.5	425.0	0.0	0.0	0.0	0.0	362.5	425.0
AREA 19	0.0	5.0	0.0	0.0	2.5	0.0	2.5	5.0
AREA 20	0.0	0.0	0.0	2.0	2.0	0.0	2.0	2.0
AREA 21	5.0	15.0	0.0	0.0	0.0	0.0	5.0	15.0

[illegible]

[illegible]

Figure VI.1 (cont'd.)

AREA 22	15.0	25.0	0.0	0.0	0.0	0.0	15.0	25.0
AREA 23	0.0	0.0	0.0	0.0	325.0	375.0	325.0	375.0
						TOTAL=	5922.0	10272.0

TOTAL P/C SERVICE PROVIDED= 16194.00 MINS.

BUDGET LEVEL TO SUPPORT THIS MIX= $ 1781.89

RHSK= 1600.0 Z(K)= 31156.8

STAFF MIX: RN: 3, LPN:15, NA:23.

STAFF/PT. CLASS

	RN		LPN		NA		TOTAL	
	A	S	A	S	A	S	A	S
AREA 1	0.0	0.0	0.0	475.0	275.0	0.0	275.0	475.0
AREA 2	37.5	11.5	0.0	0.0	0.0	338.5	37.5	350.0
AREA 3	0.0	218.4	300.0	1000.0	1000.0	1000.0	1300.0	2218.4
AREA 4	0.0	4.0	0.0	846.0	450.0	0.0	450.0	850.0
AREA 5	0.0	0.0	0.0	425.0	375.0	0.0	375.0	425.0
AREA 6	0.0	0.0	137.5	750.0	0.0	0.0	137.5	750.0
AREA 7	0.0	0.0	0.0	0.0	200.0	250.0	200.0	250.0
AREA 8	0.0	0.0	0.0	125.0	50.0	0.0	50.0	125.0
AREA 9	0.0	0.0	0.0	0.0	22.5	37.5	22.5	37.5
AREA 10	0.0	500.0	0.0	0.0	375.0	0.0	375.0	500.0
AREA 11	0.0	0.0	0.0	750.0	1000.0	1000.0	1000.0	1750.0
AREA 12	500.0	700.0	0.0	0.0	0.0	0.0	500.0	700.0
AREA 13	0.0	0.0	87.5	225.0	0.0	0.0	87.5	225.0
AREA 14	37.5	75.0	0.0	0.0	0.0	0.0	37.5	75.0
AREA 15	0.0	0.0	37.5	25.0	0.0	0.0	37.5	25.0
AREA 16	0.0	0.0	0.0	100.0	75.0	0.0	75.0	100.0
AREA 17	0.0	0.0	212.5	212.5	0.0	0.0	212.5	212.5
AREA 18	362.5	425.0	0.0	0.0	0.0	0.0	362.5	425.0
AREA 19	0.0	5.0	0.0	0.0	2.5	0.0	2.5	5.0
AREA 20	0.0	0.0	0.0	2.0	2.0	0.0	2.0	2.0

Figure VI.1 (cont'd.)

AREA 21	5.0	15.0	0.0	0.0	0.0	0.0	5.0	15.0
AREA 22	15.0	25.0	0.0	0.0	0.0	0.0	15.0	25.0
AREA 23	0.0	0.0	0.0	0.0	325.0	375.0	325.0	375.0
						TOTAL=	5884.5	9945.4

TOTAL P/C SERVICE PROVIDED= 15829.90 MINS.

BUDGET LEVEL TO SUPPORT THIS MIX= $ 1595.74

RHSK= 1400.0 Z(K)= 28027.4

STAFF MIX: RN: 8. LPN:10. NA:23.

STAFF/PT. CLASS

	RN		LPN		NA		TOTAL	
	A	S	A	S	A	S	A	S
AREA 1	0.0	0.0	0.0	475.0	275.0	0.0	275.0	475.0
AREA 2	0.0	0.0	0.0	0.0	37.5	275.0	37.5	275.0
AREA 3	0.0	0.0	150.0	1000.0	1000.0	1000.0	1150.0	2000.0
AREA 4	0.0	589.0	0.0	0.0	450.0	86.0	450.0	675.0
AREA 5	0.0	300.0	0.0	0.0	375.0	0.0	375.0	300.0
AREA 6	0.0	550.0	75.0	0.0	0.0	0.0	75.0	550.0
AREA 7	0.0	0.0	0.0	0.0	200.0	250.0	200.0	250.0
AREA 8	0.0	50.0	0.0	0.0	50.0	0.0	50.0	50.0
AREA 9	0.0	0.0	15.0	15.0	0.0	0.0	15.0	15.0
AREA 10	0.0	233.9	0.0	91.1	375.0	0.0	375.0	325.0
AREA 11	0.0	0.0	0.0	750.0	1000.0	1000.0	1000.0	1750.0
AREA 12	0.0	500.0	425.0	0.0	0.0	0.0	425.0	500.0
AREA 13	0.0	0.0	37.5	212.7	0.0	0.0	37.5	212.7
AREA 14	20.0	75.0	0.0	0.0	0.0	0.0	20.0	75.0
AREA 15	0.0	0.0	37.5	25.0	0.0	0.0	37.5	25.0
AREA 16	0.0	0.0	0.0	100.0	75.0	0.0	75.0	100.0
AREA 17	0.0	0.0	212.5	212.5	0.0	0.0	212.5	212.5
AREA 18	287.5	300.0	0.0	0.0	0.0	0.0	287.5	300.0
AREA 19	0.0	2.5	0.0	0.0	2.5	0.0	2.5	2.5

Figure VI.1 (cont'd.)

AREA 20	0.0	0.0	0.0	2.0	2.0	0.0	2.0	2.0
AREA 21	3.5	15.0	0.0	0.0	0.0	0.0	3.5	15.0
AREA 22	15.0	25.0	0.0	0.0	0.0	0.0	15.0	25.0
AREA 23	0.0	0.0	0.0	0.0	325.0	375.0	325.0	375.0
						TOTAL=	5445.5	8509.7

TOTAL P/C SERVICE PROVIDED= 13955.23 MINS.

BUDGET LEVEL TO SUPPORT THIS MIX= $ 1396.99

RHSK= 1200.0 Z(K)= 21404.6

STAFF MIX: RN: 2. LPN:12. NA:23.

STAFF/PT. CLASS	RN		LPN		NA		TOTAL	
	A	S	A	S	A	S	A	S
AREA 1	0.0	0.0	0.0	450.0	187.5	0.0	187.5	450.0
AREA 2	37.5	275.0	0.0	0.0	0.0	0.0	37.5	275.0
AREA 3	0.0	0.0	150.0	1000.0	1000.0	1000.0	1150.0	2000.0
AREA 4	0.0	0.0	0.0	181.0	382.0	494.0	382.0	675.0
AREA 5	0.0	0.0	0.0	0.0	287.5	300.0	287.5	300.0
AREA 6	0.0	0.0	75.0	0.0	0.0	550.0	75.0	550.0
AREA 7	0.0	0.0	0.0	0.0	112.5	150.0	112.5	150.0
AREA 8	0.0	0.0	0.0	50.0	25.0	0.0	25.0	50.0
AREA 9	0.0	0.0	15.0	15.0	0.0	0.0	15.0	15.0
AREA 10	0.0	0.0	0.0	325.0	225.0	0.0	225.0	325.0
AREA 11	0.0	0.0	0.0	750.0	825.0	1000.0	825.0	1750.0
AREA 12	0.0	0.0	425.0	500.0	0.0	0.0	425.0	500.0
AREA 13	0.0	0.0	37.5	125.0	0.0	0.0	37.5	125.0
AREA 14	0.0	0.0	20.0	75.0	0.0	0.0	20.0	75.0
AREA 15	0.0	0.0	20.0	7.5	0.0	0.0	20.0	7.5
AREA 16	0.0	0.0	0.0	37.5	37.5	0.0	37.5	37.5
AREA 17	0.0	0.0	62.5	62.5	0.0	0.0	62.5	62.5

Figure VI.1 (cont'd.)

AREA 18	103.1	300.0	184.4	0.0	0.0	0.0	287.5	300.0
AREA 19	0.0	2.5	0.0	0.0	1.5	0.0	1.5	2.5
AREA 20	0.0	0.0	0.0	0.5	0.5	0.0	0.5	0.5
AREA 21	3.5	7.5	0.0	0.0	0.0	0.0	3.5	7.5
AREA 22	5.0	7.5	0.0	0.0	0.0	0.0	5.0	7.5
AREA 23	0.0	0.0	0.0	0.0	250.0	325.0	250.0	325.0
						TOTAL=	4472.5	7990.5

TOTAL P/C SERVICE PROVIDED= 12463.00 MINS.

BUDGET LEVEL TO SUPPORT THIS MIX= $ 1195.21

THERE WERE 264L.P.S SOLVED AND THE TREE CONTAINED 153VERTICES

4 ROWS OF THE BOUNDING ARRAYS WERE USED

105CALLS TO POSTOP

in the objective function. The increase appears to yield not only more appropriate assignments, but also to increase the time allocated to a number of high priority care areas (viz.,4,7,10,22, and 23). It is also of note that as additional funding becomes available the majority of additional nursing time is allocated to Skilled patients. The policy implications of this phenomenon will be discussed in the next section. Finally, examination of the allocation of nursing time between high and low priority care area/classification combinations* reveals that under the exigencies of decreased funding, allocation to high priority combinations decreases at a slower rate than for low priority combinations. As an illustration, consider care area 1 for Skilled patients. The allocation remains at the upper bound (475 minutes) until funding is constrained to $1200, at which time it decreases to 450 minutes. The allocation to care area 2 for Int. A patients, on the other hand, begins to decrease at a budget level of $1600.

VI.5 Example of Model II

As the final stage in our example, we present an application of Model II (III.3). Having determined

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* See Table III.9

the effects of changes in the service level and budget constraints under the upper and lower bounds postulated in Chapter III, we would like to determine the effect of alterations in these bounds on the staff mix, nursing time allocations, assignment pattern, and necessary budget. In order to place this study in a realistic context, we will analyze the possibility of decreasing the normative bounds by various percentages in order to obtain a feasible solution at the current facility daily nursing personnel budget.

The results of the application of Model II are shown in Figure VI.2. The maximum budget was set at $900. Theta at a value of zero indicates a 30% decrease for each care area/classification lower bound, while theta at one indicates a 15% decrease. The parameter alpha at zero yields the original upper bounds, while alpha equals one implies a 15% decrease in all upper bounds. As can be observed in Figure VI.2., theta equals zero and alpha equals one yield a staff mix that may be maintained under the current facility budget. This is at the expense of a decrease in total service provided of 3055.5 minutes and a decrease in the objective function of 4128.7 units. In terms of increased budgetary

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Figure VI.2

Results of Application of Model II to Variation in the Upper and Lower Bounds

FINAL REPORT

THIS PROBLEM CONTAINED 51 CONSTRAINTS(5 ,LE,46 ,GE,46 ,EQ,) THERE WERE187 VARIABLES, 3 OF WHICH WERE INTEGER.

ALPHA STEP SIZE WAS 1.00 THETA STEP SIZE WAS 1.00

BUDGET= $ 900.00

THETA= 0.00 ALPHA= 0.00 Z= 17287.9

STAFF MIX: RN: 2,LPN: 3,NA:25,

STAFF/PT. CLASS

	RN		LPN		NA		TOTAL	
	A	S	A	S	A	S	A	S
AREA 1	0.0	0.0	0.0	315.0	131.2	0.0	131.2	315.0
AREA 2	0.0	0.0	0.0	0.0	26.2	192.5	26.2	192.5
AREA 3	0.0	97.2	0.0	302.8	897.9	1000.0	897.9	1400.0
AREA 4	0.0	0.0	0.0	0.0	450.0	472.5	450.0	472.5
AREA 5	0.0	0.0	0.0	0.0	201.2	210.0	201.2	210.0
AREA 6	0.0	0.0	52.5	0.0	0.0	385.0	52.5	385.0
AREA 7	0.0	0.0	0.0	0.0	78.7	250.0	78.7	250.0
AREA 8	0.0	0.0	0.0	0.0	17.5	35.0	17.5	35.0
AREA 9	0.0	0.0	0.0	0.0	10.5	10.5	10.5	10.5
AREA 10	0.0	0.0	0.0	0.0	375.0	227.5	375.0	227.5
AREA 11	0.0	0.0	0.0	225.0	577.5	1000.0	577.5	1225.0
AREA 12	0.0	148.7	0.0	0.0	297.5	201.3	297.5	350.0
AREA 13	0.0	0.0	26.2	87.5	0.0	0.0	26.2	87.5
AREA 14	14.0	52.5	0.0	0.0	0.0	0.0	14.0	52.5
AREA 15	0.0	0.0	14.0	5.2	0.0	0.0	14.0	5.2
AREA 16	0.0	0.0	0.0	26.2	26.2	0.0	26.2	26.2
AREA 17	0.0	0.0	43.7	43.7	0.0	0.0	43.7	43.7
AREA 18	201.2	210.0	0.0	0.0	0.0	0.0	201.2	210.0
AREA 19	0.0	1.7	0.0	0.0	1.0	0.0	1.0	1.7
AREA 20	0.0	0.0	0.0	0.3	0.3	0.0	0.3	0.3
AREA 21	2.4	5.2	0.0	0.0	0.0	0.0	2.4	5.2

Figure VI.2 (cont'd.)

APEA 22	3.5	5.2	0.0	0.0	0.0	0.0	3.5	5.2
APEA 23	0.0	0.0	0.0	0.0	325.0	375.0	325.0	375.0
						TOTAL=	3773.2	5885.5

TOTAL P/C SERVICE PROVIDED= 9658.70 MINS.

BUDGET LEVEL TO SUPPORT THIS MIX= $ 891.76

THETA= 0.00 ALPHA= 1.00 Z= 17275.9

STAFF MIX: RN: 3.LPN: 3.NA:23.

STAFF/PT. CLASS

	RN		LPN		NA		TOTAL	
	A	S	A	S	A	S	A	S
APEA 1	0.0	0.0	0.0	315.0	131.2	0.0	131.2	315.0
APEA 2	0.0	0.0	0.0	0.0	26.2	192.5	26.2	192.5
APEA 3	0.0	97.2	0.0	302.8	913.1	1000.0	913.1	1400.0
APEA 4	0.0	0.0	0.0	0.0	382.5	472.5	382.5	472.5
APEA 5	0.0	0.0	0.0	0.0	201.2	210.0	201.2	210.0
APEA 6	0.0	0.0	52.5	0.0	0.0	385.0	52.5	385.0
APEA 7	0.0	0.0	0.0	0.0	78.7	212.5	78.7	212.5
APEA 8	0.0	0.0	0.0	0.0	17.5	35.0	17.5	35.0
APEA 9	0.0	0.0	0.0	0.0	10.5	10.5	10.5	10.5
APEA 10	0.0	0.0	0.0	0.0	318.7	227.5	318.7	227.5
APEA 11	0.0	0.0	0.0	225.0	577.5	1000.0	577.5	1225.0
APEA 12	162.5	350.0	0.0	0.0	128.0	0.0	297.5	350.0
APEA 13	0.0	0.0	26.2	87.5	0.0	0.0	26.2	87.5
APEA 14	14.0	52.5	0.0	0.0	0.0	0.0	14.0	52.5
APEA 15	0.0	0.0	14.0	5.2	0.0	0.0	14.0	5.2
APEA 16	0.0	0.0	0.0	26.2	26.2	0.0	26.2	26.2
APEA 17	0.0	0.0	43.7	43.7	0.0	0.0	43.7	43.7
APEA 18	201.2	210.0	0.0	0.0	0.0	0.0	201.2	210.0
APEA 19	0.0	1.7	0.0	0.0	1.0	0.0	1.0	1.7
APEA 20	0.0	0.0	0.0	0.3	0.3	0.0	0.3	0.3

Figure VI.2 (cont'd.)

```
AREA  21    2.4    5.2    0.0    0.0    0.0    0.0    2.4    5.2
AREA  22    3.5    5.2    0.0    0.0    0.0    0.0    3.5    5.2
AREA  23    0.0    0.0    0.0    0.0  276.2  318.7  276.2  318.7

                                         TOTAL=  3615.8  5791.7

TOTAL P/C SERVICE PROVIDED=   9407.50  MINS.
BUDGET LEVEL TO SUPPORT THIS MIX= $  884.34
THERE WERE   28L.P.'S SOLVED AND THE TREE CONTAINED   17VERTICES.
   3  ROWS OF THE BOUNDING ARRAYS WERE USED
   11  CALLS TO POSTOP
INFEAS 1
PROBLEM INFEASIBLE FOR ALL THETA .GE.   1.00000
```

allocations, the proportional increase in the objective function is not as great in moving from a budget of $900 to $1200 as that for an increase from $1200 to $1400; on the other hand, the proportional increase in total service provided is greater. This would seem to indicate that relatively inappropriate assignments outweigh the increased allocation of time to high priority combinations, and it is not until further increases in the budget are made that more appropriate assignments are possible.

An interesting feature of this analysis concerns reimbursement rate policies for Medicare/Medicaid patients. At current rates, the facility may provide a total of 9407.5 minutes of patient-centered care to its patient mix, allocated as 3615.8 minutes to Int. A and 5791.7 minutes to Skilled care patients. Under an increase in personnel budget of $310.87 per day a total service level of 12463 minutes can be maintained, allocated as 4472.5 minutes to Int. A and 7990.5 minutes to Skilled care patients. The most significant increase in care provided is obviously for Skilled patients. For policy purposes, then, what would be the proportional increase in daily reimbursement rates for each level of care if it were decided to pursue the higher budget policy? Using the assumed patient mix, the increase should be apportioned as

\$1.74 per day for each Int. A patient and \$4.48 per day for each Skilled care patient. Carrying the analysis one step further, the policy of increasing personnel budget from \$1200 to \$1400 would require an additional assessment of \$2.62 per day for each Int. A patient and \$1.41 per day for each Skilled care patient. This type of result can be of great value to administrators and regulatory agencies alike, because it places in clear perspective the costs of providing additional care on a per capita basis.

VI.6 Conclusion

The sample application presented in this chapter gives an indication of the type of information made available to the facility administrator and director of nursing through use of the models and algorithms developed in this dissertation. Due to the exigencies of computer funding, the application may not have been as extensive as might have been desired; yet, with the exception of showing an example of Model II as applied to changes in the bounds on personnel, the range of possible model employment has been indicated. Use of the models to assess alternative courses of action with respect to increased service provided and budget has been stressed, but

it should be clear that the alternative application to service and budget cuts is immediate. In addition, by setting the budget constraint at various levels, the service level model can assist the administrator in recognizing suboptimal staffing patterns. Finally, further exploration into the policy aspects of the models could prove to be an interesting application of the methodology.

Chapter VII. Summary, Recommendations, and Extensions

VII.1 Introduction

This dissertation represents an effort to respond to the changing environment of and demand for long-term care. The approach taken has been essentially structure-oriented at the facility level, and is specifically aimed at providing managerial decision-makers with information that is basic for planning a nurse staffing program that responds more effectively to patient needs. Such information is obtained from the solution of two parametric representations of a normative staffing model. The model itself is based upon a patient classification system that has been derived from a comprehensive patient assessment instrument, and has been shown to bear a direct relationship to demand for nursing services. Moreover, as a direct result of the above efforts, certain theoretical advances have been made in the area of parametric mixed-integer linear programming.

In this chapter we will briefly summarize the findings of our study, provide additional insights into areas of concern, suggest procedures for model implementation, and recommend areas for further research. Finally, some possible extensions of the current effort will be presented.

VII.2 Previous Work

Several studies concerned with patient classification and nurse staffing in both acute and long-term care settings were discussed in Chapter II. It was observed that the early work of Connor [18] established the concept of patient classification based on the degree of patient independence in what are now called activities of daily living (ADL). Young's [84] regression-based pediatric patient classification system demonstrated the feasibility of using a statistical methodology to derive meaningful prediction of demand for nursing services. As was the case in both the Connor and Young studies, the classification systems developed by Poland, _et al_. [54] and the Hospital Association of New York State [30] for acute care patients attempted to link patient dependency status to nursing service demands. Typically, however, the predictions were given in terms of total nursing care time demanded for a patient mix on a nursing unit, with no specification as to the partitioning of the demand by nursing skill level.

With respect to long-term care patient classification, it was also found in most cases that the relationship between a patient's health status and a detailed specification of demand for nursing services was not provided. The classification systems of Katz,

et al. [36] and Burack [13,50] serve as typical examples of this point. Nevertheless, these two systems proved to be significant; the former for its prescient recognition of the concept of ADL, and the latter for its emphasis on goal-oriented care. The RAPIDS system of Salmon, *et al.* [61,62] addressed the issues of patient needs and placement vis-a-vis level of care; but it should be noted that acceptable results were reported only with regard to the placement question. The two studies by Parker [52,53] represent an initial effort towards the use of various statistical techniques in the derivation of long-term care patient classification. Through use of Bayes' Theorem on patient assessment data from an abbreviated list of binary health status indicators, and through the use of cluster and discriminant analysis on data derived from assessments using the CPAI, insights were obtained into the feasibility of developing effective classification systems, and key indicators, as a basis for further analysis. Finally, the Colorado study by McKnight [41,42] quantified the relationship between a tri-level patient classification system and existing patterns of nursing service care activities. Data from this study have been a key ingredient in our presentation.

Several proposed nurse staffing models were

[illegible]

surveyed, among which the mathematical programming approaches of Wolfe [80] and Liebman [38,39] were found to be most relevant to this work. Wolfe's integer programming model, although developed for acute care units, was nevertheless of interest because of its use of groupings of nursing tasks into task complexes. Although such groupings were suggested in order to reduce the model size for computational ease, they may have been too broadly drawn to allow for the incorporation of specific allocations and assignments of staff to individual care activities. Extensions to Wolfe's model proposed by Shuman [66] were shown to provide additional features that brought the original model closer to a representation of reality. The work of Liebman was presented as the only known application, previous to this dissertation, of mathematical programming techniques to long-term care personnel allocations. In this study, the insights provided as to the nature of objective function forms and coefficients proved to be invaluable; the use of psychometric measurement techniques and determinations of preferred staffing patterns (actual vs. predicted) formed a large part of the Liebman study and appeared to be most appropriate for specifying an objective function. The proposed models, however, were structured in such a way as to

[illegible]

make the presentation of alternative courses of action difficult, with a consequent increase in computational effort required in order to develop such information.

As a final note, the extensive application of computer simulation to long-term care nurse staffing, assignment, and allocation was surveyed. The studies of Turner, et al. [74], McKnight and Steorts [43], and Hundert [31] were shown to provide estimates of staffing and assignment patterns based on examination of resources predicted to meet a series of randomly generated patient demands. Although simulation models have become more sophisticated over the years in their quantification of systems, their results are subject to statistical error. It was also observed that inadequate provisions were incorporated in these models for a parametric examination of model data and model results.

VII.3 The Proposed Methodology

In retrospect, it can be seen that few of the studies surveyed in the long-term care area are capable of relating patient needs, as reflected in health status, to the provision and assignment of adequate staff to meet those needs. Recall that in Chapter III it was suggested that the statements of the administrator's staffing problem, and the aggregate budgeting

problem, given by Baloff, et al. [8] capture the essence of the type of information necessary to plan an effective staffing program and the reasons why the derivation of such information is non-trivial. The unified approach presented in this dissertation is believed to embody a rational technique for developing such information, in a manner useful to facility administrators and directors of nursing.

As the initial step in the derivation, it was demonstrated that the care area performance time data from McKnight's Colorado study [41,42] could be interpreted as performance times, per 24-hour day, for "typical" or "average" patients belonging to each of three homogeneous classification groups. Further, it was remarked that the groupings, designated as the minimum, moderate, and maximum care categories of the Colorado study, are analogous to the Intermediate B, Intermediate A, and Skilled levels of care. From these data, the set of normative average time estimates were derived.

The next step was the development of an easily applied patient classification system that could group patients into categories representing the three levels of care indicated above. Major criteria for group identification were the functioning ability and

[illegible]

psychosocial status of the patient, as well as medical status as reflected by certain medically-defined conditions. After extracting a representative sample of patient assessments based on the Collaborative Patient Assessment Instrument (CPAI) and 37 CPAI variables indicating overall patient status as defined above, a regression-based method due to Walker and Duncan [78] was applied. This method was held to be appropriate because of its ability to use an ordered, polychotomous response variable; that is, use of the appropriate level of care for a patient as adjudged by a professional panel. Further information concerning the assumptions and derivation of the Walker and Duncan method were also discussed. After analysis of the 37-variable problem, five subsets of the original variables containing 12 variables and one subset containing 9 variables were chosen for further study by recourse to results of previous studies and by backward elimination of several of the 37 variables. One such 12-variable subset was shown to possess good prediction and recognition powers, and based on comparison of several measures of performance with other subsets, was chosen as a reasonable answer to the subset selection problem. This subset, in turn, became the basis for an implementable patient classification form proposed in Chapter IV.

It was noted that this patient classification system could serve not only for nursing service demand predictions, but also as a tool for use by officials responsible for legal placement of patients.

Having thus developed a readily-applied patient level of care classification system and having shown how nursing service demands for "typical" patients of each category could be estimated, it was proposed that average unit or facility-wide demand could be predicted on a daily basis. This was accomplished by first determining the numbers of patients in each category, then multiplying each care area/classification average demand by the appropriate group membership.

Determination of an optimal nurse staffing mix and allocation and assignment patterns could then be addressed. The Basic Staffing Model (BSM) proposed in Chapter III incorporates those features recognized as being most germane to the administrator's problem. A synthesis of both appropriate assignment of nursing skill levels to care area/classification level demands and assured allocation of nursing resources to high priority demands was proposed as the basis for an objective function to be maximized. Constraints that provided for adherence to legal staffing guidelines, satisfaction of bounded demands by care area and patient classification, and recognition of the availability of nursing resources

were incorporated. The BSM was formulated as a mixed-integer linear program, and, based on the assumptions and data proposed in Chapter III, provided a 24-hour staff mix, allocation, and assignment pattern for a unit or facility.

Recognizing that presentation of a single optimal solution can be of dubious benefit to the administrator, an extension to the BSM was derived that allows for the analysis of alternative courses of action with respect to the total service provided and the personnel budget allocated. These aspects were incorporated in Model I. As an additional feature, a method for inspection of model sensitivity to the upper and lower bounds on demands as well as alterations to staffing limitations was proposed and synthesized into Model II. Models I and II were both cast as parametric mixed-integer linear programs, and algorithms for their efficient solution were derived in Chapter V. Computational results for general problems in the generic classes of both models were also reported.

As a final step in the presentation, a representative application of the methodology was given in Chapter VI. Using data obtained from an existing long-term care facility, the response of the objective function, staff mix, and allocation and assignment patterns to alterations in the total service provided,

[illegible]

[illegible]

[illegible]

personnel budget, and upper and lower bounds on demands was analyzed. In general, it was evident that the model displayed a fairly high degree of sensitivity to its parameters, thereby reiterating the necessity for analysis of this sort. The information derived in the analysis should prove to be valuable for the facility administrator and the director of nursing, both for planning a staffing program and for indicating how increased amounts of care (and their cost) can be evaluated.

VII.4 Recommendations and Extensions

Before proceeding further, a major point must be emphasized. Although the models and methodologies set forth in this dissertation are deeply rooted in concepts and data obtained from field studies of long-term care facilities, some of the proposals remain, by nature, speculative at this point. Therefore, before undertaking further theoretical work on patient classification systems and nurse staffing models, it is recommended that some form of implementation, testing, and evaluation of the work proposed herein be attempted. In this way, further experience and feedback can provide information that would be essential for extending and perhaps modifying the procedures suggested in this study.

Turning to specifics, some factors with respect to the CPAI data used in the derivation of the patient classification system are worthy of comment. The CPAI is currently undergoing revision and field-testing, a major consideration being an attempt to better quantify the psychological and social functioning status of a patient. That such factors are of importance in planning for and providing care for long-term care patients is well known. A second point is that due to a lack of data in the proper form, the special care factors listed on the last page of the CPAI* were not included in the current study. Intuitively, it would appear that both the expanded psychosocial and special care factors might affect the results presented here; the expectation is that inclusion of such factors would serve to increase the predictive capability of the model. Therefore, it is recommended that upon derivation of a representative sample of patient assessments using the revised CPAI, the analysis of Chapter IV be reapplied. Comparison of results could be highly informative.

Several recommendations concerning data used in the staffing models are in order. No new and extensive nursing activity analysis data for long-term care has

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* See Appendix A.

been produced since McKnight's Colorado study, although such an undertaking is currently underway at The Johns Hopkins University. Data obtained with respect to amounts of care provided by level of care and by existing allocation of nursing time would be invaluable as a supplement to those data made available for this dissertation. Subject to interpretation of the new results in a manner analogous to that proposed here, the models, in their current forms, would readily accept such information. As a further recommendation, data obtained by subjective estimation (e.g., appropriateness scores, upper and lower bounds on demands, and priorities) should be replicated, perhaps by more extensive use of psychometric measurement techniques such as the Q-sort.

Recall that in considering the allocation of nursing time to those tasks that are non-patient-centered, a per-shift allowance (by skill level) was subtracted from the time on duty during a normal shift. In this way, time available to devote to patient-centered activities was obtained. As a possible extension to the models, the grouping of non-patient-centered tasks into clusters similar to care areas might be attempted, with the added feature that appropriateness scores and performance times be attached to each such cluster. Assuming the availability

[illegible]

of such additional data, the proposed models could become more useful to managerial decision-makers.

A final comment concerns the inclusion of such activities as physical therapy, occupational therapy, and recreational therapy in the models. What little time was observed as being devoted to these activities in the nursing homes of the Colorado study was included in the performance times used here and allocated to nursing staff. Should additional data become available, these activities and the specialized personnel charged with their direction could easily be included by adding appropriate variables and constraints in extended versions of our models.

In retrospect, the unified approach to patient classification and nurse staffing presented in this dissertation represents a first step towards relating patient health status to the managerial decisions faced by facility administrators and directors of nursing. Innovations in meaningful and readily applied classification and staffing models have been stressed, as well as demonstration of the methodology. It is believed that such an approach will prove itself in meeting the changes in the long-term care system almost certain to occur in succeeding years.

Appendix A

Collaborative Patient Assessment Instrument

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PATIENT CLASSIFICATION FORM

DATE OF CLASSIFICATION	
INTERVIEWER	
RESEARCH GROUP	
PATIENT NUMBER	

SOCIO-DEMOGRAPHIC DATA

1. SOCIAL SECURITY NUMBER (Specify) ____

2. BIRTHDATE (Specify) ____ Month ____ Day ____ Year

3. BIRTHPLACE (Check One)
   - __ USA. SPECIFY STATE ____
   - __ OTHER. SPECIFY COUNTRY ____

4. SEX (Check One)
   - __ MALE
   - __ FEMALE

5. RACE (Check One)
   - __ WHITE __ AMER. INDIAN __ CHINESE __ OTHER (SPECIFY) ____
   - __ NEGRO __ JAPANESE __ FILLIPINO

6. MARITAL STATUS (Check One)
   - __ SINGLE __ WIDOWED __ SEPARATED
   - __ MARRIED __ DIVORCED HOW MANY YEARS? ____

7. RELIGIOUS PREFERENCE (Check One)
   - __ NONE __ JEWISH __ OTHER (SPECIFY) ____
   - __ CATHOLIC __ PROTESTANT

8. RESIDENCE ADDRESS ____ Number ____ Street ____ City ____ State ____ Zip Code

9. PATIENT LOCATION (PRESENT) (Check One)

   RESIDENTIAL
   - __ PRIVATE RESIDENCE
   - __ RENTED ROOM(S) (COMMERCIAL)

   HEALTH CARE FACILITY
   - __ DOMICILIARY/PERSONAL CARE
   - __ INTERMEDIATE CARE
   - __ NURSING HOME
   - __ EXTENDED CARE
   - __ CHRONIC DIS./REHAB. HOSP.
   - __ MENTAL HOSPITAL
   - __ OTHER SPECIALTY HOSP.
   - __ SHORT TERM ACUTE HOSP.

10. LENGTH OF TIME AT LOCATION

    IF RESIDENTIAL: SPECIFY ____ YRS. ____ MOS.

    IF HEALTH CARE FACILITY: SPECIFY DATE OF: THIS ADMISSION ____ FIRST ADMISSION ____

11. LIVING ARRANGEMENTS (Check Those Which Apply)
    - __ OWN HOME
    - __ ANOTHER'S HOME
      - __ PAYING __ NOT PAYING
    - __ RENTED ROOM(S)
    - __ ALONE
    - __ WITH SPOUSE
    - __ WITH OTHER. WHO? ____
    - __ HEALTH RELATED FACILITY (SPECIFY TYPE) ____

12. LIVING CHILDREN SPECIFY BY NUMBER: SONS: ____ DAUGHTERS: ____

13. EDUCATION (Check Those Which Apply)
    - __ NO SCHOOLING
    - __ SPECIAL EDUCATION. SPECIFY YEARS ____
    - __ GRADES COMPLETED. SPECIFY ____
    - __ HIGH SCHOOL DIPLOMA
    - __ TRADE, TECHNICAL, VOCATIONAL SCHOOLING
    - __ SOME UNDERGRADUATE COLLEGE. SPECIFY YRS. ____
    - __ COLLEGE COMPLETED
    - __ GRADUATE COLLEGE

14. USUAL OCCUPATION SPECIFY: ____ OR CHECK ONE: __ MILITARY __ HOUSEWIFE __ NEVER EMPLOYED

15. EMPLOYMENT STATUS (Check Those Which Apply)
    - __ EMPLOYED
    - __ RECEIVING PAY
    - __ RETIRED
    - __ UNEMPLOYED
    - __ WORKING, HRS./WEEK ____
      - __ INSIDE HOME/FACILITY
      - __ OUTSIDE HOME/FACILITY
    - __ RECEIVING PENSION
    - __ NEVER IN LABOR MARKET

16. FAMILY INCOME (Check One)
    - __ LESS THAN $3,000 __ $3,000 - $4,999 __ $5,000 - $6,999
    - __ $7,000 - $9,999 __ $10,000 - $14,999 __ $15,000 PLUS

17. HEALTH CARE COVERAGE (Check Those Which Apply)
    - __ NONE
      - __ SELF-PAY
      - __ NO CHARGE
    - __ MEDICARE
    - __ MEDICAID

    LIST ALL OTHER TYPES OF HEALTH INSURANCE PATIENT HAS
    ____
    ____
    ____

DIAGNOSIS(ES) (List)
____ ____
____ ____
____ ____
____ ____

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INTERVIEWER		DATE OF CLASSIFICATION					
RESEARCH GROUP		PATIENT NUMBER					

FUNCTIONING STATUS ITEMS

18. MOBILITY LEVEL (Check and Complete as Applicable)
GOES OUTSIDE HOUSE/FACILITY
___W/O HELP
___WITH HELP, DESCRIBE ______
MOVES ABOUT HOUSE/FACILITY
___W/O HELP
___WITH HELP, DESCRIBE ______
CONFINED
___CHAIR
___BED
DESCRIBE ANY DEVICES USED ______

19. TRANSFERRING (Check and Complete as Applicable)
TRANSFERS
___W/O HELP
___WITH HELP, DESCRIBE ______
DOES NOT TRANSFER
___DONE BY OTHERS
___BEDFAST
DESCRIBE ANY DEVICES USED ______

20. WALKING (Check and Complete as Applicable)
WALKS
___W/O HELP
___WITH HELP, DESCRIBE ______
DOES NOT WALK
___BEDFAST
___CHAIRFAST
DESCRIBE ANY DEVICES USED ______

21. WHEELING (Check and Complete as Applicable)
WHEELS
___W/O HELP
___WITH HELP, DESCRIBE ______
DOES NOT WHEEL
___WALKS
___CHAIRFAST
___BEDFAST
___IS WHEELED BY OTHERS
DESCRIBE ANY DEVICES USED ______

22. STAIR CLIMBING (Check and Complete as Applicable)
GOES UP AND DOWN STAIRS
___W/O HELP
___WITH HELP, DESCRIBE ______
DOES NOT CLIMB STAIRS
___GOES UP/DOWN CURB
___GOES UP/DOWN ONE/TWO STEPS
___USES RAMP FOR ONE/TWO STEPS
___USES ELEVATOR/CHAIR LIFT
DESCRIBE ANY DEVICES USED ______

23. BATHING (Check and Complete as Applicable)
BATHES
___W/O HELP
___WITH HELP, DESCRIBE ______
___IS BATHED BY OTHERS
WHERE
___BED
___SINK
___TUB
___SHOWER
DESCRIBE ANY DEVICES USED ______

24. DRESSING (Check and Complete as Applicable)
DRESSES
___W/O HELP
___WITH HELP, DESCRIBE ______
___IS DRESSED BY OTHERS
KIND OF DRESS
___STREET CLOTHES
___ROBE & PJ/GOWN
___SLIPPERS
___SHOES
DESCRIBE ANY DEVICES USED ______

25. EATING/FEEDING (Check and Complete as Applicable)
EATS
___W/O HELP
___WITH HELP, DESCRIBE ______
WHERE
___BED
___CHAIR IN ROOM
___DINING ROOM
IS FED
___ORALLY BY OTHERS
___TUBE FED
___PARENTERALLY FED
DESCRIBE ANY DEVICES USED ______

26. TOILETING (Check and Complete as Applicable)
USES TOILET ROOM
___W/O HELP
___WITH HELP, DESCRIBE ______
WHEN
___ALL THE TIME
___DAY ONLY
___NEVER
SUBSTITUTES
___BEDPAN/URINAL
___COMMODE
DESCRIBE ANY DEVICES USED ______

27. BOWEL FUNCTION (Check and Complete as Applicable)
___NO PROBLEM
___IMPACTION
___INVOLUNTARY LOSS
OSTOMY
___SELF CARE
___NOT SELF CARE
DESCRIBE ANY CONTROL PROGRAM BEING USED ______

28. BLADDER FUNCTION (Check and Complete as Applicable)
___NO PROBLEM
___RETENTION
___INVOLUNTARY LOSS
___OSTOMY
___INDWELLING CATHETER
___EXTERNAL DEVICE, DESCRIBE ______
___SELF CARE
___NOT SELF CARE
DESCRIBE ANY CONTROL PROGRAM BEING USED ______

29. ORIENTATION (Check One and Complete as Applicable)
___ORIENTED
___DISORIENTED
___COMATOSE
IF DISORIENTED USE THE TABLE TO INDICATE BY (✓) THE AREA(S) AND FREQUENCY.

	ALWAYS	PART-TIME
TIME		
PLACE		
PERSON		

30. COMMUNICATION OF NEEDS (Check One and Complete as Applicable)
___VERBALLY
___NONVERBALLY, SPECIFY HOW: ______
___DOES NOT COMMUNICATE

31. BEHAVIOR PATTERN (Check and Complete as Applicable)
___APPROPRIATE BEHAVIOR
___INAPPROPRIATE BEHAVIOR
___WANDERING, PASSIVE
___ABUSIVE, AGGRESSIVE
___OTHER, DESCRIBE ______
___ONCE A WEEK OR LESS
___MORE OFTEN THAN ONCE A WEEK

M-14B

INTERVIEWER		DATE OF CLASSIFICATION	
RESEARCH GROUP		PATIENT NUMBER	

IMPAIRMENTS

32. SIGHT (Check and Complete as Applicable)
___NO IMPAIRMENT
___LEGALLY BLIND
___IMPAIRMENT, DESCRIBE ________
TYPE OF COMPENSATION
___GLASSES
___CONTACT LENS
___LARGE PRINT
___OTHER, DESCRIBE ________

33. HEARING (Check and Complete as Applicable)
___NO IMPAIRMENT
___DOES NOT HEAR
___IMPAIRMENT, DESCRIBE ________
TYPE OF COMPENSATION
___LOUD VOICE
___SHOUTING
___HEARING AID
___LIP READING
___OTHER, DESCRIBE ________

34. SPEECH (Check and Complete as Applicable)
___NO IMPAIRMENT
___DOES NOT SPEAK
___IMPAIRMENT, DESCRIBE ________
TYPE OF COMPENSATION
___WRITES
___GESTURES
___SIGN LANGUAGE
___OTHER, DESCRIBE ________

35. FRACTURES AND DISLOCATIONS (Check and Complete as Applicable)
___NONE
___HIP FRACTURE
___WITH PROSTHESIS
___WITH REPAIR
___OTHER FRACTURE(S), DESCRIBE ________
___DISLOCATION(S), DESCRIBE ________

36. JOINT MOTION (Check One and Complete as Applicable)
___NO IMPAIRMENT ___IMPAIRMENT, USE TABLE TO SPECIFY SITE, JOINT AND SIDE(S) AND CHECK (✓) COLUMN FOR TYPE

JOINT SITE (R, L, B)	PAIN & SWELLING	LIMITED MOBILITY	IMMOBILITY	INSTABILITY

37. MISSING LIMBS (Check One and Complete as Applicable)
___NONE MISSING ___MISSING, USE TABLE TO SPECIFY MISSING PART(S) AND CHECK (✓) COLUMN FOR PROSTHESIS

MISSING PART(S) RT., LEFT, BOTH	PROSTHESIS

38. PARALYSIS (Check One and Complete as Applicable)
___NONE
___PARALYSIS, DESCRIBE TYPE AND LOCATION
___MUSCULAR WEAKNESS, WASTING OR ATROPHY

39. DENTITION (Check and Complete as Applicable)
___NO TEETH MISSING
___SOME TEETH MISSING
___EDENTULOUS
TYPE OF COMPENSATION
___PARTIAL PLATE
___COMPLETE UPPER PLATE
___COMPLETE LOWER PLATE
___OTHER APPLIANCE, DESCRIBE ________
___SPECIAL DIET, DESCRIBE ________
USES FOR CHEWING
___YES
___NO

MEDICAL STATUS ITEMS

40. RISK FACTOR MEASUREMENTS Specify the Readings and Dates in the Spaces Provided Below:

	READING	DATE		READING	DATE
HEIGHT			BLOOD CHOLES.		
WEIGHT			BUN		
BLOOD PRESSURE			ALBUMINUREA		

41. CIGARETTE SMOKING (Check One and Complete as Applicable)
___NEVER SMOKED ___EX-SMOKER ___PRESENT SMOKER
SPECIFY NUMBER PER DAY: (PAST/PRESENT) ______

42. MEDICALLY DEFINED CONDITIONS (Check either No or Yes Column for each condition listed. Also specify as indicated the Information for Each Condition in the Last Two Columns Below:)

	NO	YES	TYPE/LOCATION (AS APPLICABLE)	DURATION (YRS./MOS.)
ALCOHOLISM				
ANEMIA				
ANGINA/MI				
ARTHRITIS				
CARDIAC ARRHYTHMIA				
CONGESTIVE HT. FAILURE				
DECUBITUS ULCERS				
DIABETES MELLITUS				
DRUG ABUSE				
HYPERTENSION				
MALIGNANCY				
MENTAL ILLNESS				
NEUROLOGICAL DISORDERS				
RESPIRATORY DISEASE(CHR.)				

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INTERVIEWER		DATE OF CLASSIFICATION	
RESEARCH GROUP		PATIENT NUMBER	

SPECIAL CARE FACTORS (Check and Complete as Applicable)

ITEM	SPECIFY TYPE (AS APPLICABLE)	FREQUENCY (1xDAY, 1xWEEK, ETC.)
___ DECUBITI CARE		
___ DRESSINGS (STERILE, LARGE, BULKY)		
___ INHALATION THERAPY (IPPB-O_2-NEBULIZER)		
___ IRRIGATIONS (EXCLUDING BLADDER CATHETER) SPECIFY LESION		
___ OSTOMY CARE SPECIFY SITE OF OSTOMY		
___ MEDICATIONS (ORAL, PARENTEROL, TOPICAL, ETC.)		
___ DIVERSIONAL THERAPY (SOCIAL, RECREATIONAL ACTIVITIES)		
___ SUPPORTIVE MEASURES - EMOTIONAL REASSURANCE		
___ PATIENT TEACHING (TRAINING)		
___ SPECIAL DIET THERAPY		
___ SPECIAL LAB TESTS	NOT IN CLASSIFICATION	
___ THERAPIES (PT, OT, SPEECH)		
___ PODIATRY CARE		
___ PSYCHO-THERAPY		
___ SUCTIONING		
___ CONVULSIONS AND SEIZURES		
___ APPLICATION OF HEAT AND COLD		
___ INTAKE-OUTPUT		
___ FORCE FLUIDS		
___ CATHERIZATION		
___ GROOMING		
___ RESTORATIVE-REHABILITATIVE NURSING (ROM)		
___ POSITIONING		
___ SUPPORT BANDAGES		
___ HAS STRONG PREFERENCES & HABIT PATTERNS		
___ RESTRAINTS		
___ NEW ADMISSION		
___ PREPARATION FOR DISCHARGE		
___ OTHER, DESCRIBE		

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Thesis
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